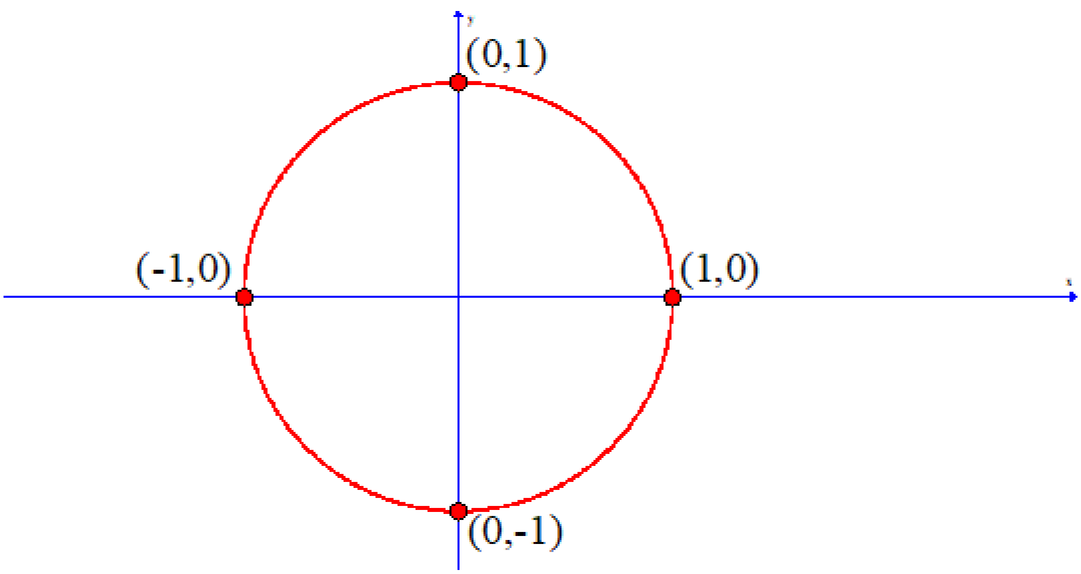
Trigonometric RatiosSolutions

Use the following information to answer the first question.

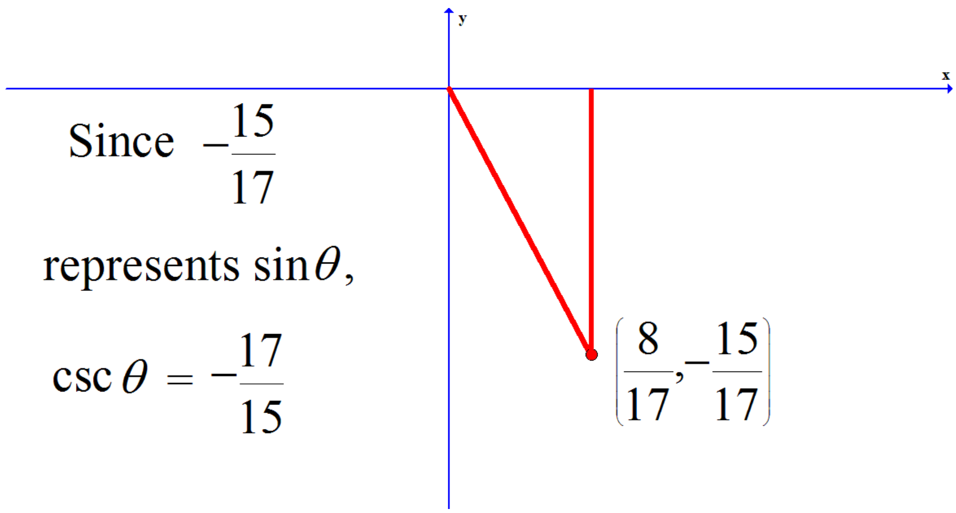
When working with exact trigonometric ratios of 1, -1, or 0, noting the coordinates on the axes of the unit circle, can be useful. Recalling that for any point on the unit circle, P(θ) = (cos θ, sin θ) and tan θ = $\frac{\sin\theta}{\cos\theta}$, answer the following questions without a calculator.



1. What is
   a) sin 270^0 = -1
   b) cos 90^0 = 0
   c) tan 360^0 = 0
   d) sin $-\pi$ = 0
   e) cos $\left(-\frac{\pi}{2}\right)$ = 0
   f) tan 3π = 0
   g) sec 180^0 = -1
   h) csc $\left(\frac{3\pi}{2}\right)$ = -1
   i) cot 450^0 = 0

2. On a unit circle, Point P$\left(\frac{8}{17}, -\frac{15}{17}\right)$ lies on the terminal arm of an angle in standard position. What is the exact value of cscθ?

a) $\frac{15}{8}$ b) $\frac{17}{15}$ c) $-\frac{17}{15}$ d) $-\frac{17}{8}$



Since $-\frac{15}{17}$ represents $\sin\theta$,

$\csc\theta = -\frac{17}{15}$

3. The terminal arm of θ, when drawn in standard position, contains point M(x,y), where M is on the unit circle. If cos θ = $-\frac{6}{11}$, and tan θ < 0, what is the value of y?

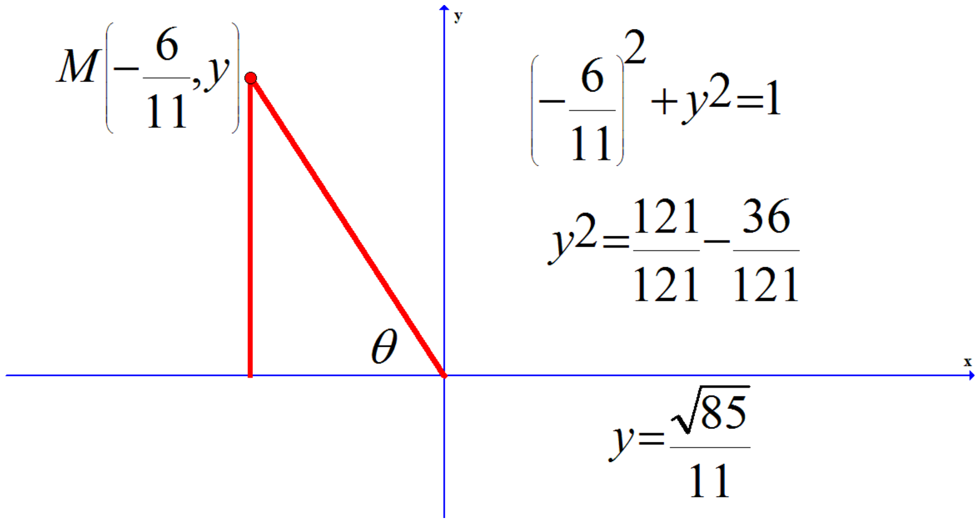
a) $\frac{\sqrt{85}}{11}$ Ans b) $-\frac{\sqrt{85}}{11}$ c) $\frac{85}{6}$ d) $-\frac{85}{6}$

Since both cosine and tangent are negative, Point M is in quadrant 2.

Given the fact that cos θ = $-\frac{6}{11}$, the x-coordinate of Point M is $-\frac{6}{11}$.

Use the equation of a circle, $x^2 + y^2 = 1$, to find the value for y.

See the following diagram.



$$M\left(-\frac{6}{11}, y\right)$$

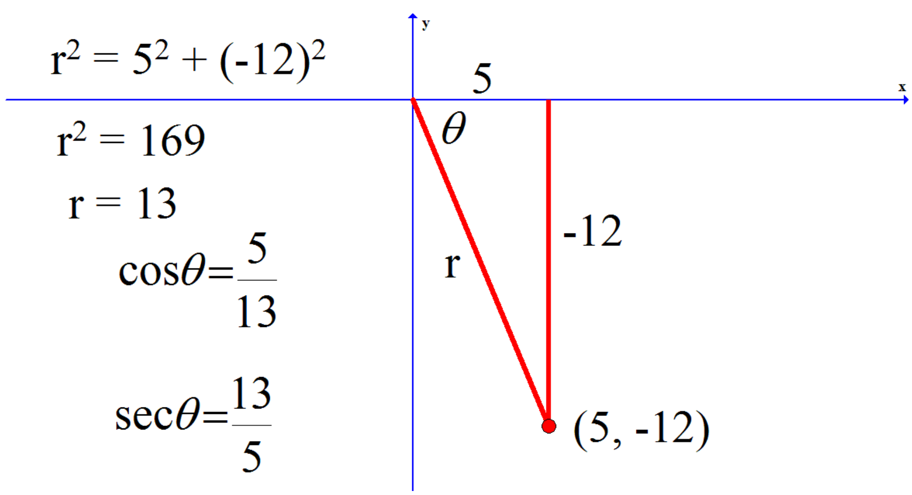
$$\left(-\frac{6}{11}\right)^2 + y^2 = 1$$

$$y^2 = \frac{121}{121} - \frac{36}{121}$$

$$y = \frac{\sqrt{85}}{11}$$

4. The point D(5,-12) lies on the terminal arm of an angle θ in standard position. What is the exact value of sec θ? Show a diagram.

The exact value of secθ is 13/5.



$$r^2 = 5^2 + (-12)^2$$

$$r^2 = 169$$

$$r = 13$$

$$\cos\theta = \frac{5}{13}$$

$$\sec\theta = \frac{13}{5}$$

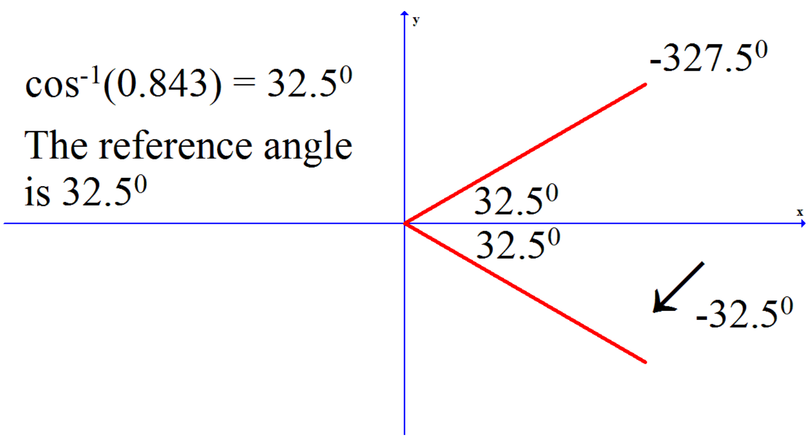
5. Determine the measures of all angles that satisfy each of the following and use diagrams.
   a) cos θ = 0.843 in the domain $-360^0 < \theta < 180^0$. Give approximate answers to the nearest tenth.

Use the calculator to determine the reference angle, of 32.5^0.

Consider that the domain is $-360^0 < \theta < 180^0$. The cosine ratio is positive in quadrants 1 and 4.

The first positive angle (32.5) is within the domain. The second positive ratio would be 360 – 32.5 or 327.5. However, this cannot be a solution because it is not in the domain.

Rotating clockwise (or negative rotation) will produce 2 solutions in the domain. The first solution is -32.5 and the second is -327.5(32.5 – 360).



The 3 solutions are 32.5^0, -32.5^0 and -327.5^0.

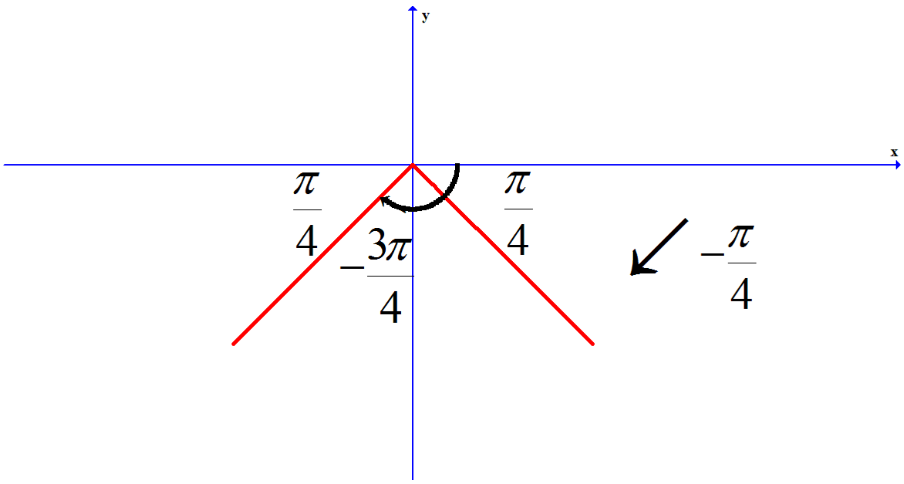
   b) $\csc\theta = -\frac{2}{\sqrt{2}}$ in the domain $-2\pi \le \theta \le \pi$. Give exact answers.

Change the ratio from a reciprocal to a primary ratio equivalent.

$\sin\theta = -\frac{\sqrt{2}}{2}$.

Consider that the domain is $-2\pi \le \theta \le \pi$.

Sin is negative in quadrants 3 and 4. Use the calculator to determine that the reference angle is $\frac{\pi}{4}$.



Within the stated domain, there are 2 solutions, both measured in negative radians.

The first solution is $-\frac{\pi}{4}$ and the second solution is $-\frac{3\pi}{4}$.

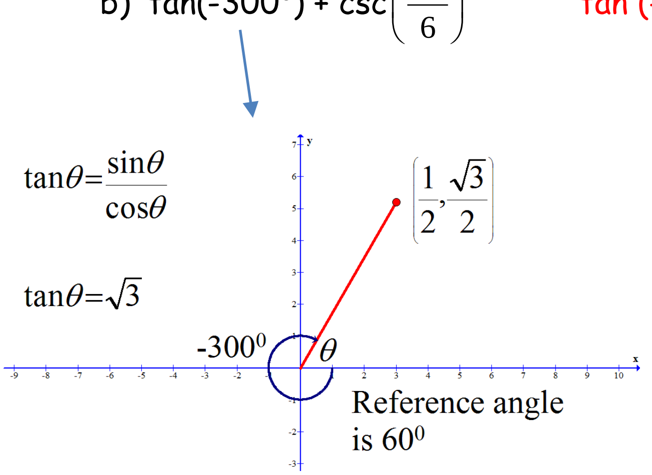
6. Determine the exact values for each of the following:

   a) $\tan\left(\frac{\pi}{2}\right)$ Undefined

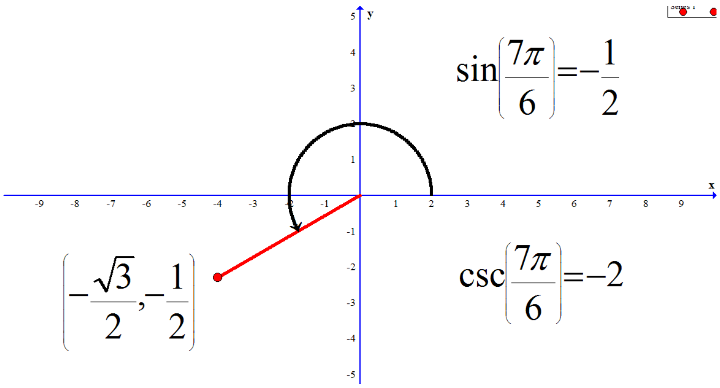
   b) $\tan(-300^0) + \csc\left(\frac{7\pi}{6}\right)$ tan $(-300^0) = \sqrt{3}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\tan\theta = \sqrt{3}$



For the second part, csc$\left(\frac{7\pi}{6}\right)$, find the sin of $\left(\frac{7\pi}{6}\right)$ reciprocate the answer.



csc$\left(\frac{7\pi}{6}\right)$ = -2

tan(-300^0) + csc$\left(\frac{7\pi}{6}\right)$

= $\sqrt{3}$ + -2

= $\sqrt{3}$ - 2

c) sin$\left(\frac{3\pi}{4}\right)$ - tan^2 (-45^0)

$\left(\frac{3\pi}{4}\right)$ is in quadrant 2 and the sin ratio is positive. It relates to the special triangle,

45-45-90, where the sin is $\frac{\sqrt{2}}{2}$.

-45^0 is in quadrant 4, and the tan ratio is negative. It relate to the same special triangle, 45-45-90, where tan is (-1).

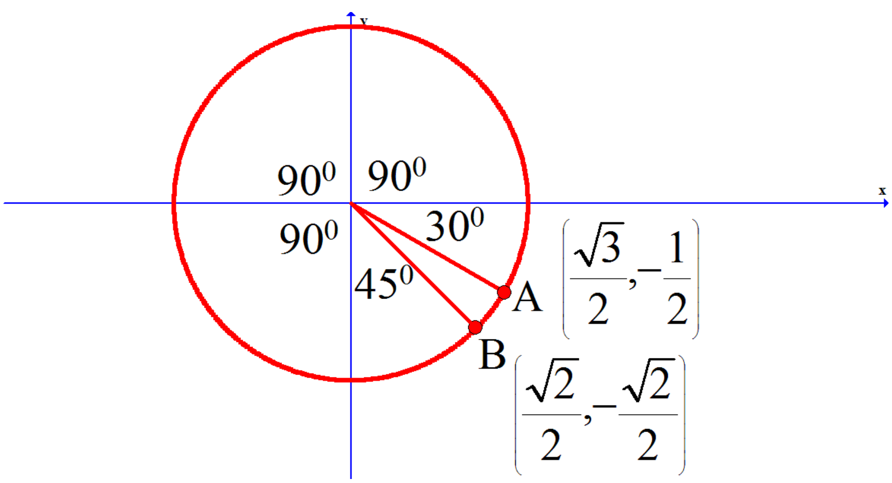
Squaring (-1) and subtracting it from $\frac{\sqrt{2}}{2}$, results in an exact value of $\frac{\sqrt{2}}{2}$ - 1.

The exact value of sin$\left(\frac{3\pi}{4}\right)$ - tan^2 (-45^0) is $\frac{\sqrt{2}}{2}$ - 1.

Use the following information to answer the next question.

Points A$\left(\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$ and B$\left(\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$ are 2 points on the unit circle. The Point O(0,0) is the centre of the unit circle.

7. The measure of the largest angle, AOB, in degrees, is __345__.



90 + 90 + 90 + 30 + 45 = 345

8. The Point K($\frac{1}{2}$, y) is on the terminal arm of angle θ drawn in standard position on the unit circle. An angle that could be co-terminal with θ is

a) 300^0 b) 135^0 c) 120^0 d) 30^0

On the unit circle, the first coordinate is the cosine of the angle and the second coordinate is the sine of the angle.

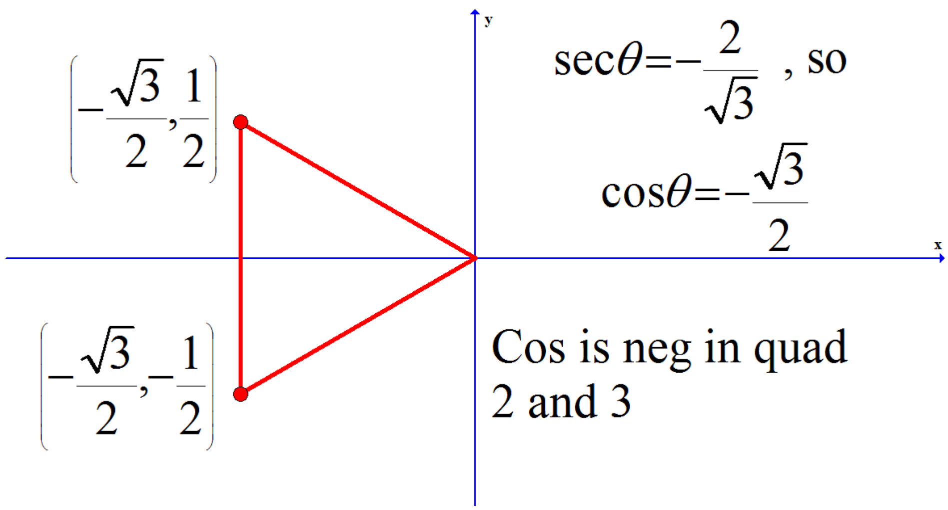
The angle created at point K has a cosine ratio of $\frac{1}{2}$.

Of the given choices, 300^0 is the only angle having a cosine ratio of $\frac{1}{2}$.

9. If sec θ = $-\frac{2}{\sqrt{3}}$, where $0 \le \theta < 2\pi$, then θ lies in quadrants __i__ and tan θ is equal to __ii____.

The statement above is completed by the information in row **D**

Row	i	ii
A	2 and 4	$\pm\frac{1}{\sqrt{3}}$
B	2 and 3	$\pm\sqrt{3}$
C	2 and 4	$\pm\sqrt{3}$
D	2 and 3	$\pm\frac{1}{\sqrt{3}}$



$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \pm\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$

$= \pm\left(\frac{1}{2}\right)\left(\frac{2}{\sqrt{3}}\right)$

$= \pm\frac{1}{\sqrt{3}}$

10. Given $\cos\theta = \frac{\sqrt{13}}{7}$, where $\frac{3\pi}{2} \le \theta \le 2\pi$, determine the exact value of $\cot\theta$.

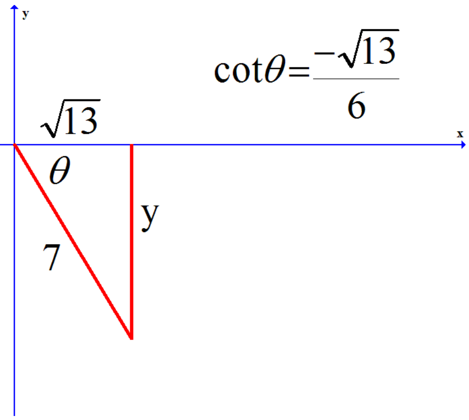
We are given a positive cosine ratio. Cosine is positive in quadrants 1 and 4.

By, $\frac{3\pi}{2} \le \theta \le 2\pi$, we are told that θ is in quadrant 4. In order to find cotθ, we need to determine all the sides in the right angle triangle.

$$7^2 - \left(\sqrt{13}\right)^2 = y^2$$

$$49 - 13 = y^2$$

$$y = -6$$



When using Pythagorean Theorem to solve for y above, the value of y could be positive or negative. In this case it is negative, because in quadrant 4, the y values are negative.

The exact value of cotθ is $-\dfrac{\sqrt{13}}{6}$

11. If $\tan \theta = \dfrac{4}{3}$, where $0 \le \theta < 2\pi$, then the largest possible value of θ, to the nearest tenth, is __4.1__ radians.

Tan is positive in quadrants 1 and 3. Use the calculator (in radians) to find the reference angle. $\text{Tan}^{-1}(4/3) = 0.93$. Rotating a half a circle, or π radians, from there will determine the largest possible value in the stated domain.

$$\tan^{-1}\left(\frac{4}{3}\right) = 0.93$$