The Binomial Theorem Solutions

1. Given ${}_{19}C_k\,(2x)^v\left(-\frac{1}{4}\right)^5$, which is one term in the expansion of $\left(2x-\frac{1}{4}\right)^n$, what is the value of

   i) k? ii) v? iii) n? iv) the term # of ${}_{19}C_k\,(2x)^v\left(-\frac{1}{4}\right)^5$?

i) The value of 'k' and the exponent on the second term of the binomial are always equal.

The value of 'k' is 5.

ii) The 2 exponents applied to the 2 terms of the binomials always add to n. In this question, n = 19. Thus, v + 5 = 19.

The value of v is 14.

iii) Given the component, ${}_{19}C_k$, n is the subscripted number before the C.

The value of n is 19.

iv) The term number is one more than the value of k.

The term number of ${}_{19}C_k\,(2x)^v\left(-\frac{1}{4}\right)^5$ is 6.

Use the following information to answer the next question.

Given $(a + b)^6$, consider the following statements.	
Statement 1	The total number of terms is 5.
Statement 2	For the term, $20a^3b^m$, m =3
Statement 3	The sum of the coefficients of all the terms is 64.
Statement 4	If b is an integer, the constant term is 6b.

2. The 2 correct statements are _<u>2</u>_ and _<u>3</u>_.

Statement 1 is false. There is one more term than the value of n. There are 7 terms.

Statement 2 is true. The exponents on the two terms, 'a' and 'b', must add to n, or 6 in this case.

Statement 3 is true.

${}_6C_0$	+	${}_6C_1$	+	${}_6C_2$	+	${}_6C_3$	+	${}_6C_4$	+	${}_6C_5$	+	${}_6C_6$
1	+	6	+	15	+	20	+	15	+	6	+	1

= 64

Statement 4 is false. The constant is b^6.

3. In the expansion of $(2x - y^3)^{11}$, the coefficient of the term containing x^3y^{24} is

a) 1320 b) -1320 c) 2480 d) -2480

The second term of the binomial is $(-y^3)$. In order to have a term containing y^{24}, the exponent outside of the brackets of $(-y^3)$, has to be 8 [since 3 X 8 = 24]. The exponent of 8 on this second term is the same as the value of k.

${}_{11}C_8\,(2x)^3(-y^3)^8$

$= \quad (165)(8x^3)(y^{24})$

$= \quad 1320x^3y^{24}$

The coefficient of the term containing x^3y^{24} is 1320.

4. The constant term in the expansion of $\left(5x^5 + \frac{1}{x^2}\right)^{14}$ can be written in the form , **abcabc**. The values of a, b, and c, are respectively, _6_, _2_, and _5_.

When determining a constant term, the first step is finding the value of k that will result in a term of x^0 [x^0 is equal to 1; which means that there are no lettered components in the term. This is the meaning of a constant term].

If there are any coefficients in any of the terms, they can be removed as they have no bearing on the first step.

With the exponent in the denominator, move the term to the numerator, and change the sign on the exponent.

$(x^5)^{14-k} (x^{-2})^k = x^0$

$(x^{70-5k}) (x^{-2k}) = x^0$

When multiplying 2 powers with the same base, keep the base and add the exponents.

$x^{70-7k} = x^0$

Drop the base of x, and set the exponents equal to each other, and solve for k.

70 – 7k = 0.

70 = 7k

10 = k

Step 2 is to now use the value of 'k' in the general term to find the constant term.

$_{14}C_{10}(5x^5)^4(\frac{1}{x^2})^{10}$

$(1001)\ (625x^{20})\ (\frac{1}{x^{20}})$

= 625625

The values of A, B, and C are 625.

5. Find the middle term of $\left(x^4 - \frac{1}{\sqrt{3x^2}}\right)^8$.

In order for a middle term to exist, there must be an odd number of terms. Since the exponent on the binomial is 8, and there is always 1 more term than this number, there are 9 terms. Therefore, this expansion has a middle term.

1 2 3 4 Middle 6 7 8 9

The 5^{th} term is the middle term. For the 5^{th} term, the value of k is 4 [always one less].

${}_8C_4(x^4)^4\left(-\frac{1}{\sqrt{3x^2}}\right)^4$

$= 70x^{16}\left(\frac{1}{9x^4}\right)$

$= \frac{70x^{12}}{9}$

The middle term of $\left(x^4 - \frac{1}{\sqrt{3x^2}}\right)^8$ is $\frac{70x^{12}}{9}$.

6. A term in the expansion of $(ax - 2y)^9$ is $-225\ 792x^2y^7$. What is the value of a?

In order to have a term with y^7, we know that the exponent on the second term of the binomial, i.e. (-2y), must be 7. Thus, we know that k = 7.

${}_9C_7(ax)^2(-2y)^7 = -225\ 792x^2y^7$

$36(a^2x^2)(-(2)^7(y)^7) = -225\ 792x^2y^7$

$-4608(a^2x^2)(y^7) = = -225\ 792x^2y^7$

Divide both side by $-4608x^2y^7$

$a^2 = 49$

$a = 7$

The value of a is 7.

7. Find the coefficient of the 3rd term of $(2x + \sqrt{2})^5$.

For the 3rd term, k = 2.

${}_5C_2(2x)^3(\sqrt{2})^2$

$= \quad (10)(8x^3)(2)$

$= \quad 160x^3$

The coefficient of the 3rd term is 160.

8. The 4th term of $\left(x - \frac{1}{2}\right)^n$ is $-15x^7$. Determine the value of n.

In order to get a term with x^7, the exponent on the first term in the binomial must be 7. For the 4th term, k = 3. This means that the exponent on the second term of the binomial must be 3. The sum of these two exponents is equal to n.

The value of n is 10.

9. Which term number of $\left(x^2 - \frac{1}{x}\right)^6$ is the constant term? What is the value of this constant term?

The first step in finding a constant term is to find the value of 'k' that would produce an x^0. Since $x^0 = 1$, the letter is eliminated; thus we end up with a constant term.

Re-write letters in the denominator as an equivalent by moving the power to the numerator. The sign on the exponent will then change. The coefficient of (-1) in the second term has no bearing on our determining the value of 'k'.

$(x^2)^{6-k}(x^{-1})^k = x^0$

$(x^{12-2k})(x^{-k}) = x^0$

When multiplying two powers with the same base, keep the base and add the exponents.

$x^{12-3k} = x^0$

Drop the bases and set the exponents equal, and solve for 'k'.

$12 - 3k = 0$

$12 = 3k$

$4 = k$

Step 2 is using the value of 'k' in the general term to find the constant term.

With k = 4, we are looking for the 5th term.

$_6C_4(x^2)^2\left(\frac{-1}{x}\right)^4$

$= \quad (15)(x^4)\left(\frac{1}{x^4}\right)$

= 15

The constant of 15 is the 5th term.

													Sum of Row
						1							$2^0 = 1$
					1		1						$2^1 = 2$
				1		2		1					$2^2 = 4$
			1		3		3		1				$2^3 = 8$
		1		4		6		4		1			$2^4 = 16$
	1		5		10		10		5		1		$2^5 = 32$
1		6		15		20		15		6		1	$2^6 = 64$

Pascal's Triangle

10. a) Fill in the next row of Pascal's Triangle.
    b) What is the sum of the 9th row? $2^8 = 256$
    c) Suppose a Pizza Restaurant advertized a number, which represents all the ways someone could build a pizza using at least 1 topping, to a maximum of 6 toppings. What is this number? Which row of the triangle would be closest to this number? Why is the row number slightly different from the advertized number?

This would be row 7. The sum of row 7 is 2^6 or 64. However, the advertized number would be 63, because the add says **at least 1**, which then eliminates ${}_6C_0$ (which is 1) from the total.

11. The expansion of $(2x + 3)^{a-5}$ has 7 terms.

a) What is the value of a?

With 7 terms, the value of the exponent, a – 5, must be equal to 6 (1 less than the number of terms)

$a - 5 = 6$

$a = 11$

The value of 'a' is 11.

b) What is the coefficient of the 1st term?

${}_6C_0(2x)^6(3)^0$

$(1)(64x^6)(1)$

The coefficient of the first term is 64.

c) What is the constant term?

The constant term is 3^6, or 729.