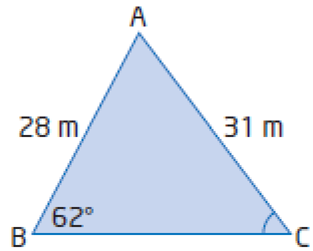
<u>The Ambiguous Case Practice</u>

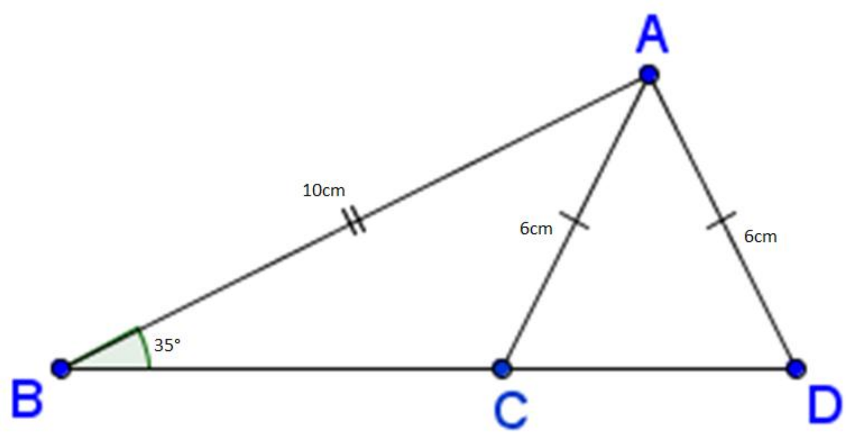
Use the following information to answer the first question.

Consider the triangle below.



1. Which statement is correct?
   A) There are 2 triangles possible because c < b.
   B) There are 2 triangles possible because c < c sin B.
   C) There is only 1 triangle possible because b > c.
   D) There is only 1 triangle possible because c sin B > c.

Use the diagram below to answer the next question.



2. The largest of the 2 options, to the nearest degree, for the angle opposite the 10 cm side is _____.

3. Which statement below is true?

A) For the ambiguous case in triangle ABC, when angle A is an obtuse angle, if a > b, then there is one solution.

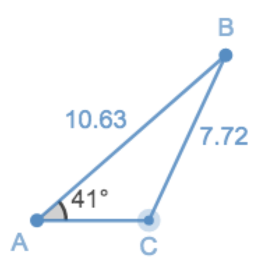
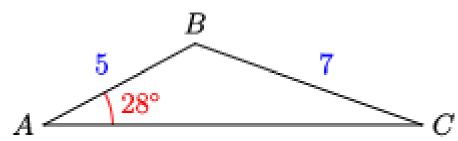
B) For the ambiguous case in triangle ABC, when angle A is an obtuse angle, if a = b, then there is one solution.

C) For the ambiguous case in triangle ABC, when angle A is an obtuse angle, if a < b, then there is one solution.

D) For the ambiguous case in triangle ABC, when angle A is an obtuse angle, it is possible to have 2 solutions.

Use the following information below to answer the next question.

Consider the following triangles.

<u>Triangle I</u>	<u>Triangle II</u>
	

4. One of the two triangles above has 2 possible answers for angle B. State which triangle has two possible answers and determine their values.

5. How many distinct triangles can be formed is $\angle A = 35^0$, a = 10 and b = 13?
   A) 0 B) 1 C) 2 D) 3

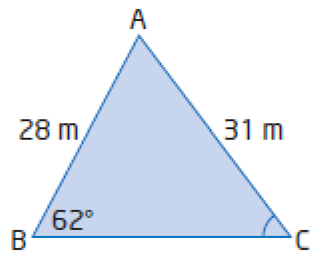
6. In ΔABC, $\angle A = 74^0$, a = 59.2 and c = 60.3. What are the two possible values for $\angle C$, to the nearest tenth?
   A) 73.7^0 and 106.3^0
   B) 73.7^0 and 163.7^0
   C) 78.3^0 and 101.7^0
   D) 78.3^0 and 168.3^0

7. Solve the triangle if $\angle A = 38^0$, a = 40 and b = 52. Express angles and side lengths to the nearest tenth.

The Ambiguous Case Practice Solutions

Use the following information to answer the first question.

Consider the triangle below.



1. Which statement is correct?

   A) There are 2 triangles possible because $c < b$.

   B) There are 2 triangles possible because $c < c \sin B$.

   C) There is only 1 triangle possible because $b > c$.

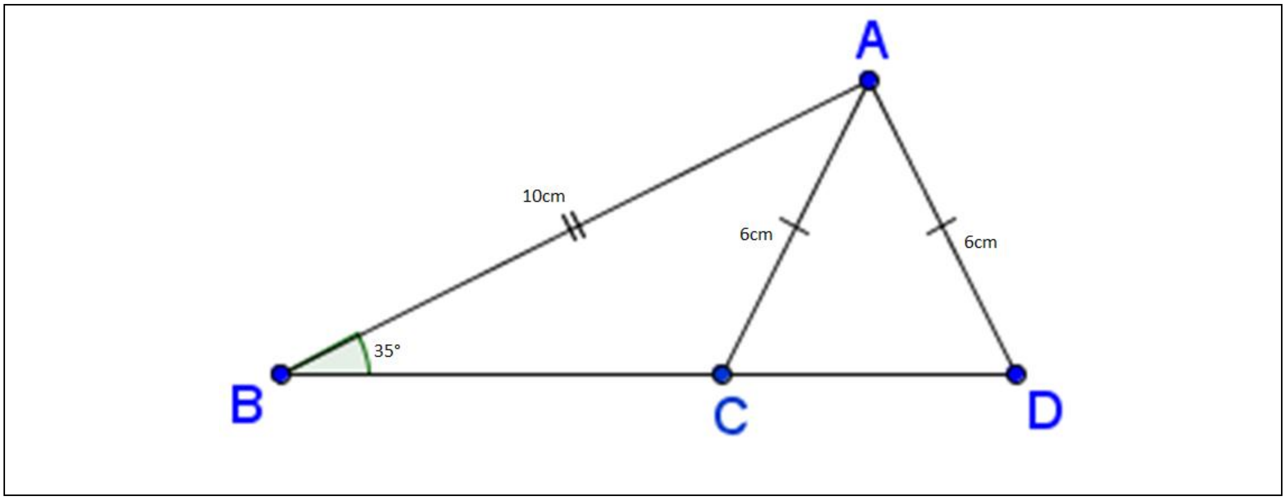
   D) There is only 1 triangle possible because $c \sin B > c$.

Solution

When the side opposite the angle is greater than the other given side, only one triangle is possible.

The correct answer is C.

Use the diagram below to answer the next question.



2. The largest of the 2 options, to the nearest degree, for the angle opposite the 10 cm side is _107^0_.

Solution

We will determine the acute angle D first.

$$\frac{\sin D}{10} = \frac{\sin 35}{6}$$

$$\sin D = \frac{(\sin 35)(10)}{6}$$

sin D = 0.9559…

$\sin^{-1}(0.9559\ldots) \approx 72.9^0$

To the nearest degree, angle D is 73^0.

Since angle D and angle C are supplementary, angle C is $180^0 - 73^0$, or 107^0.

The largest of the 2 options, to the nearest degree, for the angle opposite the 10 cm side is _107^0_.

3. Which statement below is true?

   A) For the ambiguous case in triangle ABC, when angle A is an obtuse angle, if a > b, then there is one solution.

   B) For the ambiguous case in triangle ABC, when angle A is an obtuse angle, if a = b, then there is one solution.

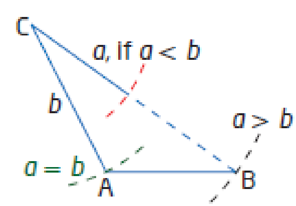
   C) For the ambiguous case in triangle ABC, when angle A is an obtuse angle, if a < b, then there is one solution.

   D) For the ambiguous case in triangle ABC, when angle A is an obtuse angle, it is possible to have 2 solutions.

Solution

For the ambiguous case in △ABC, when ∠A is an obtuse angle:

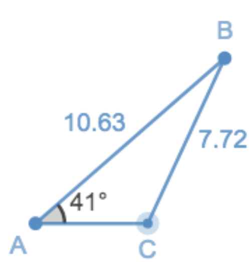
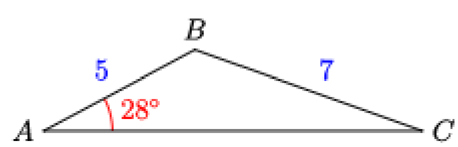
- $a \leq b$ no solution
- $a > b$ one solution



The correct answer is A.

Use the following information below to answer the next question.

Consider the following triangles.

Triangle I	Triangle II
	

4. One of the two triangles above has 2 possible answers for angle B. State which triangle has two possible answers and determine their values.

Solution

In triangle II, a > c, so there is only one triangle possible.

For triangle I, determine h, which is the length of the opposite side if there is a right angle triangle.

$$\sin 41 = \frac{h}{10.63}$$

h = (sin 41)(10.63)

h = 6.9739…

Since a is larger than h but less than c, there are two triangles possible.

h < a < c

6.9739… < 7.72 < 10.63

$$\frac{\sin C}{10.63} = \frac{\sin 41}{7.72}$$

$$\sin C = \frac{(\sin 41)(10.63)}{7.72}$$

sin *C* = 0.9033…

$\sin^{-1}(0.9033\ldots) \approx 65^0$

[Note: In the diagram, angle C is obtuse, but we have found the acute angle that satisfies the same criteria for this triangle. Angle C is actually the supplement of 65^0, or 115^0]

For one triangle, the 3 angles are $41^0 + 65^0 + B$.

For the other triangle, the 3 angles are $41^0 + 115^0 + B$.

Knowing that the 3 angles must add to 180^0, in one triangle B is equal to $180^0 - (41^0 + 65^0)$ or 74^0.

In the other triangle, B is equal to $180^0 - (41^0 + 115^0)$ or 24^0.

Triangle I has two possible values for angle B, which are 74^0 and 24^0.

5. How many distinct triangles can be formed is $\angle A = 35^0$, a = 10 and b = 13?

A) 0 B) 1 C) 2 D) 3

Solution

Since a < b, and a > than (sin35)(b), there are two possible triangles.

An alternative way to think about this would be to first find the angle of the side opposite the side of length 13.

$$\frac{\sin B}{13} = \frac{\sin 35}{10}$$

$$\sin B = \frac{(\sin 35)(13)}{10}$$

Angle B = 48^0.

Now determine the supplement of 48^0, which is 132^0.

Now, if the original angle of 35^0 is added to each of these two angles, and their sum is less than 180^0, we know there are two triangles.

$35^0 + 48^0 < 180^0$

$35^0 + 132^0 < 180^0$

Therefore, there must be two triangles.

6. In ΔABC, $\angle A = 74^0$, a = 59.2 and c = 60.3. What are the two possible values for $\angle C$, to the nearest tenth?
   A) 73.7^0 and 106.3^0
   B) 73.7^0 and 163.7^0
   C) 78.3^0 and 101.7^0
   D) 78.3^0 and 168.3^0

Solution

$$\frac{59.2}{\sin 74} = \frac{60.3}{\sin C} \quad 180 - 78.3 = 101.7$$

$$C \approx 78.3$$

The correct answer is C.

7. Solve the triangle if $\angle A = 38^0$, a = 40 and b = 52. Express angles and side lengths to the nearest tenth.

Solution

Use the sine law to find angle B.

$$\frac{\sin 38^\circ}{40} = \frac{\sin B}{52}$$

$$\frac{0.6157}{40} \approx \frac{\sin B}{52}$$

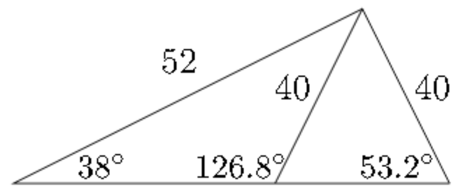
$$52 * \frac{0.6157}{40} \approx \sin B$$

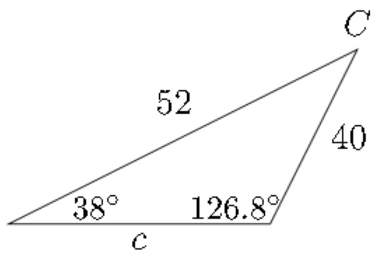
$$0.8004 \approx \sin B$$

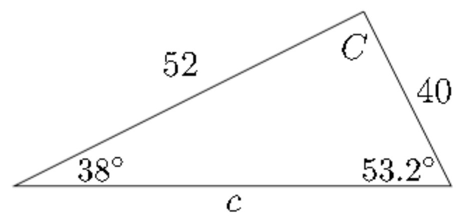
$\sin^{-1}(0.8004) = 53{,}2^0$

Since a < b, and a > (sin38)(52), we know there are two possible triangles. The other possible angle opposite to the side with length 52, is the supplement of 53.2^0, or 126.8^0.

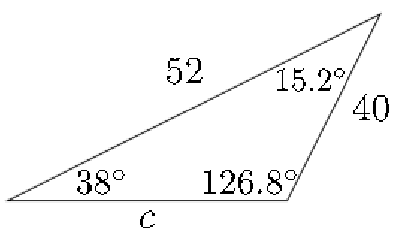
See the diagram below.

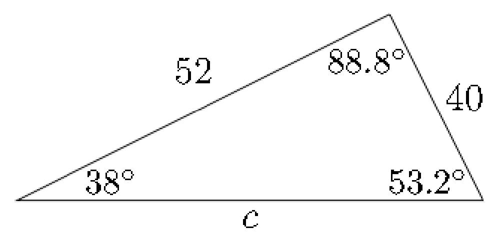






The two values for angle C can now be determined since we know that the 3 angles in a triangle must add to 180^0.





To find the two possible lengths for side C, use the sine law appropriate for each triangle.

$$\frac{40}{\sin 38^\circ} = \frac{c}{\sin 15.2^\circ}$$
$$\frac{40}{0.6157} \approx \frac{c}{0.2622}$$
$$0.2622 * \frac{40}{0.6157} \approx c$$
$$17.0 \approx c$$

$$\frac{40}{\sin 38^\circ} = \frac{c}{\sin 88.8^\circ}$$
$$\frac{40}{0.6157} \approx \frac{c}{0.9998}$$
$$0.9998 * \frac{40}{0.6157} \approx c$$
$$65.0 \approx c$$

The two possible solutions are:

$$\angle A = 38^\circ \qquad a = 40$$
$$\angle B \approx 126.8^\circ \qquad b = 52$$
$$\angle C \approx 15.2^\circ \qquad c \approx 17.0$$

$$\angle A = 38^\circ \qquad a = 40$$
$$\angle B \approx 53.2^\circ \qquad b = 52$$
$$\angle C \approx 88.8^\circ \qquad c \approx 65.0$$