

Solving Maximum/Minimum Problems

Practice Questions

1. The "Looking Good" company currently sells one of its skin care products for \$20 each, with 1500 being the average sales per month. You have been hired to determine if the current price is yielding the best profit for the company. You decided to conduct a survey of current and potential buyers. Your results indicate that sales will increase by 100 for each \$1 reduction in the cost of the product.
 - a) What cost for the skin care product will result in the highest profit?
 - b) What will the highest profit be in one month?

2. A local hockey team regularly fills the 1200 seat arena and charges \$6/ticket. A survey indicates that for every \$0.5 (50 cent) increase in price, 50 fewer fans will attend the games. What price per ticket will result in a maximum revenue and what is the maximum revenue?

3. One number is 9 more than the other. Find the numbers such that the difference between the square of the larger and 3 times the square of the smaller is a maximum.

4. The Stark family have 150 m of fencing to make a rectangular run. If they use the side of the house as one side of the run, what dimensions will give the maximum area?

5. The coach of a college football team would like his punter to punt the ball a little bit higher. If he could do that, the ball would have more hang time and allow the tacklers more time to get down field. His current maximum height is 50 yards. The coach is helping the punter change the angle of contact of the ball. With this new angle, the quadratic equation that models the path of the ball is, $y = -0.036x^2 + 2.88x + 2.4$. How much higher should the ball go with this new angle, compared to his current height?

6. If the sum of the base and height of a triangle must be 12 cm, how long should you make the base to give a maximum area?

7. The weekly profit, P , of an oil refinery is given by:

$P(x) = -0.04x^2 + 560x$, where x is the number of barrels of oil refined. Next month, after the installation of new equipment, the company would like to see their maximum weekly profit increased by 7%. What will the maximum weekly profit be next month?

Solving Maximum/Minimum Problems

Practice Questions Answers

1. The "Looking Good" company currently sells one of its skin care products for \$20 each, with 1500 being the average sales per month. You have been hired to determine if the current price is yielding the best profit for the company. You decided to conduct a survey of current and potential buyers. Your results indicate that sales will increase by 100 for each \$1 reduction in the cost of the product.
- What cost for the skin care product will result in the highest profit?
 - What will the highest profit be in one month?

$$\text{Revenue} = (\text{cost per product}) (\text{number of sales})$$

Let x = the number of \$1 reductions

$$\begin{aligned}\text{Revenue} &= (20 - x)(1500 + 100x) \\ &= 30\,000 + 2000x - 1500x - 100x^2 \\ &= -100x^2 + 500x + 30\,000 \\ &= -100(x^2 - 5x) + 30\,000 \\ &= -100\left(x^2 - 5x + \frac{25}{4} - \frac{25}{4}\right) + 30\,000 \\ &= -100\left(x - \frac{5}{2}\right)^2 + 625 + 30\,000 \\ &= -100\left(x - \frac{5}{2}\right)^2 + 30\,625\end{aligned}$$

The highest profit

There are $2\frac{1}{2}$ \$1 reductions,

Resulting in a product price of \$17.50.

2. A local hockey team regularly fills the 1200 seat arena and charges \$6/ticket. A survey indicates that for every \$0.5 (50 cent) increase in price, 50 fewer fans will attend the games. What price per ticket will result in a maximum revenue and what is the maximum revenue?

$$\text{Revenue} = (\text{number of tickets}) (\text{cost per ticket})$$

Let x = the number of \$0.5 increases

$$\begin{aligned} \text{Revenue} &= (1200 - 50x) (6 + 0.5x) \\ &= 7200 + 600x - 300x - 25x^2 \\ &= -25x^2 + 300x + 7200 \\ &= -25(x^2 - 12x + 36 - 36) + 7200 \\ &= -25(x^2 - 12x + 36) + 900 + 7200 \\ &= -25(x - 6)^2 + 8100 \end{aligned}$$

The maximum revenue

There are 6, \$0.5 increases, resulting
in a ticket price of \$9.

3. One number is 9 more than the other. Find the numbers such that the difference between the square of the larger and 3 times the square of the smaller is a maximum.

Let the first number = x

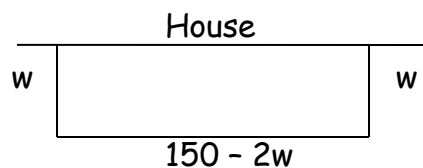
Let the second number = $x + 9$

$$\begin{aligned} \text{Maximum} &= (x + 9)^2 - 3x^2 \\ &= x^2 + 18x + 81 - 3x^2 \\ &= -2x^2 + 18x + 81 \end{aligned}$$

$$\begin{aligned}
&= -2(x^2 - 9x) + 81 \\
&= -2\left(x^2 - 9x + \frac{81}{4} - \frac{81}{4}\right) + 81 \\
&= -2\left(x - \frac{9}{2}\right)^2 + 40.5 + 81 \\
&= -2\left(x - \frac{9}{2}\right)^2 + 121.5
\end{aligned}$$

The first number is 4.5, and
The second number is 13.5 (4.5 + 9)

4. The Stark family have 150 m of fencing to make a rectangular run. If they use the side of the house as one side of the run, what dimensions will give the maximum area?



$$\begin{aligned}
\text{Area} &= w(150 - 2w) \\
&= 150w - 2w^2 \\
&= -2(w^2 - 75w) \\
&= -2\left(w^2 - 75w + \frac{5625}{4} - \frac{5625}{4}\right) \\
&= -2\left(w - \frac{75}{2}\right)^2 + 2812.5
\end{aligned}$$

The dimensions are 37.7 m, and
150 - 2(37.5), or, 75 m.

5. The coach of a college football team would like his punter to punt the ball a little bit higher. If he could do that, the ball would have more hang time and allow the tacklers more time to get down field. His current maximum height is 50 yards. The coach is helping the punter change the angle of contact of the ball. With this new angle, the quadratic equation that models the path of the ball is, $y = -0.036x^2 + 2.88x + 2.4$. How much higher should the ball go with this new angle, compared to his current height?

$$\begin{aligned}
 \text{Maximum height} &= -0.036x^2 + 2.88x + 2.4 \\
 &= -0.036(x^2 - 80x) + 2.4 \\
 &= -0.036(x^2 - 80x + 1600 - 1600) + 2.4 \\
 &= -0.036(x - 40)^2 + 57.6 + 2.4 \\
 &= -0.036(x - 40)^2 + 60
 \end{aligned}$$


 The maximum height.

Compared to the punter's current maximum height, the ball should travel 10 yards higher.

6. If the sum of the base and height of a triangle must be 12 cm, how long should you make the base to give a maximum area?


$$\text{Base} + \text{Height} = 12$$

$$\text{Let } h = h$$

$$\text{Let } b = 12 - h$$

$$\begin{aligned}
 \text{Area of a triangle} &= \frac{(b)(h)}{2} \\
 &= \frac{(12 - h)(h)}{2}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{-h^2 + 12h}{2} \\
&= \left(\frac{-1}{2}\right)h^2 + 6h \\
&= \left(\frac{-1}{2}\right)(h^2 - 12h) \\
&= \left(\frac{-1}{2}\right)(h^2 - 12h + 36 - 36) \\
&= \left(\frac{-1}{2}\right)(h - 6)^2 + 18
\end{aligned}$$



 The height will be 6, which makes
 the base 6 (12 - 6) as well.

7. The weekly profit, P , of an oil refinery is given by:

$P(x) = -0.04x^2 + 560x$, where x is the number of barrels of oil refined. Next month, after the installation of new equipment, the company would like to see their maximum weekly profit increased by 7%. What will the maximum weekly profit be next month?

$$\begin{aligned}
P(x) &= -0.04x^2 + 560x \\
&= -0.04(x^2 - 14\,000x) \\
&= -0.04(x^2 - 14\,000x + 49\,000\,000 - 49\,000\,000) \\
&= -0.04(x - 7000)^2 + 1\,960\,000
\end{aligned}$$

The maximum weekly profit is currently \$1 960 000. With the 7% increase, the new maximum weekly profit should be \$2 097 200.