Solving Logarithm and Exponential Equations Solutions

1. Solve $5^{x-3} = 1700$ to 2 decimal places.

Take the log of both sides of the equal sign.

$\log 5^{x-3} = \log 1700$

Using the Power Law of logarithms, move the exponent in front of the log.

$(x - 3)\log 5 = \log 1700$

Multiply to remove the brackets.

$x\log 5 - 3\log 5 = \log 1700$

Isolate the term with 'x'.

$x\log 5 = \log 1700 + 3\log 5$

Divide both sides by log5

$$x = \frac{(\log 1700 + 3\log 5)}{\log 5}$$

$x = 7.62$

2. Which step below will lead to the solution of $6^{3x+1} = 8^{x+3}$?

a) $\frac{3\log 8 + \log 6}{3\log 6 + \log 8}$ b) $\frac{3\log 8 - \log 6}{3\log 6 - \log 8}$ c) $\frac{\log 8 + \log 6}{\log 6 + 3\log 8}$ d) $\frac{3\log 8 - \log 6}{\log 6 + \log 8}$

Take the log of both sides of the equal sign.

$\log 6^{3x+1} = \log 8^{x+3}$

Using the Power Law of logarithms, move the exponent in front of the log.

$(3x+1)\log 6 = (x+3)\log 8$

Multiply to remove the brackets.

$3x\log 6 + \log 6 = x\log 8 + 3\log 8$

Gather 'x' terms to one side and non 'x' terms to the other side.

$3x\log 6 - x\log 8 = 3\log 8 - \log 6$

Factor out 'x'.

$x(3\log 6 - \log 8) = 3\log 8 - \log 6$

$$x = \frac{3\log 8 - \log 6}{3\log 6 - \log 8}$$

3. There are 2 solutions to the logarithmic equation $\log_7(x - 3)^2 = 2$. The sum of these 2 solutions is

a) 1 b) 3 c) 6 d) 10

Convert from logarithmic form to exponential form.

$7^2 = (x - 3)^2$

$49 = (x - 3)^2$

$49 = x^2 - 6x + 9$

Set equal to zero and solve.

$0 = x^2 - 6x - 40$

$0 = (x - 10)(x + 4)$

$x = 10$ or -4

The sum of 10 and -4 is 6.

Use the laws of logarithms to solve the next 2 questions.

4. Solve $\log_2(11 + x) + \log_2(x - 1) = 6$, and identify any extraneous roots.

Whenever there is a logarithmic equation in which multiple terms have the same base, with addition or subtraction signs within the equation, it is very likely that the product and/or quotient laws will be applied. In this case, the 2 terms on the left can be simplified into 1 term by using the product law of logarithms.

$\log_2(11 + x)(x - 1) = 6$

Converting to exponential form would be the next step.

$2^6 = (11 + x)(x - 1)$

$64 = x^2 + 10x - 11$

Set equal to zero and solve.

$0 = x^2 + 10x - 75$

$0 = (x + 15)(x - 5)$

Using the zero principle, $x = -15$ or $x = 5$.

<u>Verify $x = 5$</u>

$\log_2(11 + (5)) + \log_2((5) - 1) = 6$

$\log_2 16 + \log_2 4 = 6$

$4 + 2 = 6$

$6 = 6$

Thus, 5 is a solution.

If -15 is substituted into the equation, the result would be taking the log of a negative number. Since by definition, this is not possible, we say that -15 is an extraneous root.

The solution is 5 and -15 is the extraneous root.

5. Solve $\log_3(2x^2 - 2x) = \log_3(x - 1) + 2$

In this question, there are 2 logarithmic terms, one on each side of the equal sign. The first step is to move the logarithmic terms so they are both on the same side of the equal sign. In this case, it would be most efficient to subtract $\log_3(x - 1)$ from both sides.

$\log_3(2x^2 - 2x) - \log_3(x - 1) = 2$

Combine the two logarithmic terms into one term using the quotient law.

$$\log_3\left(\frac{2x^2 - 2x}{x - 1}\right) = 2$$

The value of the power can be simplified through factoring.

$$\log_3\left(\frac{2x(x-1)}{x - 1}\right) = 2$$

= $\log_3 2x = 2$

= $3^2 = 2x$

= $9 = 2x$

= $4.5 = x$

<u>Verify x = 4.5</u>

$\log_3(2(4.5)^2 - 2(4.5)) = \log_3((4.5) - 1) + 2$

$\log_3(31.5) = \log_3(3.5) + 2$

$\log_3(31.5) - \log_3(3.5) = 2$

$$\log_3\left(\frac{31.5}{3.5}\right) = 2$$

$\log_3 9 = 2$

$2 = 2$

6. Radioisotopes are used to diagnose various illnesses. Iodine-131 (I-131) is administered to a patient to diagnose thyroid gland activity. The original dosage contains 280MBq of I-131. If none is lost from the body, then after 6 hours there are 274MBq of I-131 in the patient's thyroid. What is the half-life of I-131, to the nearest day?

$$274 = 280\left(\frac{1}{2}\right)^{\frac{6}{x}}$$

In many exponential questions, the format is similar. We begin with a certain quantity. In this case it is 280MBq. We then do something to it by way of a constant (double it, triple it, cut it in half, etc.). In this case, because it is a half-life question, the constant is $\frac{1}{2}$. The exponent represents how many times the starting amount is halved. The exponent here is a fraction. Remember, that the bottom number of the fraction tells us how long it takes for the constant to apply itself. In other words, the 'x' we are trying to find is the half-life. The 274 is the final amount.

To solve, first isolate the power. Here, both sides are divided by 280.

$$\frac{274}{280} = \left(\frac{1}{2}\right)^{\frac{6}{x}}$$

Take the log of both sides and then move the exponent in front of the log using the Power Law of logarithms.

$$\log\left(\frac{274}{280}\right) = \left(\frac{6}{x}\right)\log\left(\frac{1}{2}\right)$$

$$= \qquad \frac{\log\left(\frac{274}{280}\right)}{\log\left(\frac{1}{2}\right)} = \frac{6}{x}$$

Cross multiply to find the value of x. The value of x is 192 hours. Divide 192 by 24 hours. **The half-life is 8 days.**

7. The compound interest formula is $A = P(1 + i)^n$, where A is the future amount, P is the present amount or Principal, i is the interest rate per compounding period expressed as a decimal, and n is the number of compounding periods. Danita inherits $15 000 and invests in a guaranteed investment certificate (GIC) that earns 3.6% compounded quarterly (every 3 months). How long will it take for the GIC to grow to $17 000?

A = 17 000

P = 15 000

i = The yearly rate of 3.6% divided by 4 (quarterly). $\frac{3.6\%}{4} = 0.9\%$ (as a decimal, 0.009)

n = the unknown we are trying to determine.

$17\ 000 = 15\ 000\ (1.009)^n$ [n will represent the number of yearly quarters]

Divide both sides by 15 000 to isolate the power.

$$\frac{17}{15} = 1.009^n$$

Take the log of both sides and move the exponent 'n' to the front of the log.

$$\log\left(\frac{17}{15}\right) = n \log 1.009$$

Divide both sides by log1.009

$$\frac{\log \frac{17}{15}}{\log 1.009} = n$$

n = 13.97

Remember that n represents the number of yearly quarters. So, $\frac{13.97}{4} \cong 3.5$

It will take approximately 3.5 years for the GIC to grow to $17 000.

8. The largest lake lying entirely within Canada is Great Bear Lake, in the Northwest Territories. On a summer day divers find that the light intensity is reduced by 6% for every 2 metres below the water surface. To the nearest tenth of a metre, at what depth is the light intensity 20% of the intensity at the surface?

A reduction of 6% means that the initial starting number, and all subsequent numbers, will be multiplied by (100% - 6%, or 94%). This number expressed as a decimal is 0.94.

Assuming that the starting amount is 100%, the equation is: $20 = 100(0.94)^x$, where 'x' represents the number of 2 m increments.

Divide both sides by 100 to isolate the power:

$0.2 = (0.94)^x$

Take the log of both sides and move the exponent in front of the log.

$\log 0.2 = x\log(0.94)$

$$\frac{\log 0.2}{\log 0.94} = x$$

$26 = x$

There are 26, 2 m increments, so the depth of the water is 52 m.

9. Solve $\log_2 \sqrt{x+4} = \frac{5}{2}$

Convert from log form to exponential form.

$$2^{\frac{5}{2}} = \sqrt{x+4}$$

Square both sides to remove the radical sign.

$$\left(2^{\frac{5}{2}}\right)^2 = \left(\sqrt{x+4}\right)^2$$

$2^5 = x + 4$

$32 = x + 4$

$28 = x$

10. Solve for x, given $2^{\frac{x}{3}} = 18$

Take the log of both sides and move the exponent to the front of the log.

$$\left(\frac{x}{3}\right)\log 2 = \log 18$$

$$\frac{x}{3} = \frac{\log 18}{\log 2}$$

$$x = \frac{3\log 18}{\log 2}$$

$x \cong 12.51$

Use the following information to answer the next question.

Consider the following steps in solving $\log_5 x - \log_5(x - 2) = 3$

Step 1 $\log_5\left(\frac{x}{x-2}\right) = 3$

Step 2 $5^3 = \left(\frac{x}{x-2}\right)$

Step 3 $125x - 2 = x$ [This should be $125x - 250$]

Step 3 $124x = 2$

Step 5 $x = \frac{1}{62}$

11. Identify the error and determine the correct solution.

Step 3 should be $125x - 250 = x$

$124x = 250$

$x = \frac{250}{124} = \frac{125}{62}$

12. Given, $m\log_p n + 7 = k$, express n^m in terms of p and k.

Subtract 7 from both sides and move the 'm' to the exponential position by applying the Power Law of logarithms.

$\log_p n^m = k - 7$

Convert from log form to exponential form.

$p^{k-7} = n^m$

13. When $5m^2 = k$, is expressed in log form, the result is

a) $\log_m\left(\frac{5}{k}\right) = 2$

b) $\log_2\left(\frac{5}{k}\right) = m$

c) $\log_2\left(\frac{k}{5}\right) = m$

d) $\log_m\left(\frac{k}{5}\right) = 2$ Answer

Divide both sides by 5 to isolate the power.

$m^2 = \left(\frac{k}{5}\right)$

Convert to logarithmic form.

The answer is $\log_m\left(\frac{k}{5}\right) = 2$

14. If $\log_c k = 2$, then what is the value of $\log_c \sqrt[4]{k}$?

Quite often, when there are two steps, as there are in this question, re-arranging the first part, leads to a substitution in the second part.

Convert the first part of the question to exponential form.

$c^2 = k$

The equivalent of the second part is $\log_c k^{\frac{1}{4}}$

Now substitute for k in the first part, for k in the second part.

$\log_c (c^2)^{\frac{1}{4}}$

= $\log_c c^{\frac{1}{2}}$

Recalling that $\log_b b^n = n$,

$\log_c c^{\frac{1}{2}} = \frac{1}{2}$

15. If $m^2 = 10$, determine the value of c in, $\log_c(m+1) + \log_c(m-1) = 2$.

In the logarithmic equation, combine the two terms on the left into one term.

$\log_c(m + 1)(m - 1) = 2$

$\log_c(m^2 - 1) = 2$

Substitute 10 for m^2.

$\log_c(10 - 1) = 2$

$c^2 = 9$

$c = 3$