Polynomials

Polynomials are expressions having 1 or more terms.

Terms can be just numbers, just letters or numbers and letters multiplied together. Addition and subtraction signs separate terms.

A **binomial** is a polynomial expression having 2 terms.

A **trinomial** is a polynomial expression having 3 terms.

Coefficients are numbers in front of the letters in any given term.

The **degree** of any **term** is the sum of the exponents of all letters in that term.

The **degree** of a **polynomial** is the highest degree of any term.

A polynomial may have a **constant** term. A **constant** term is just a number without a letter.

Practice

Polynomial	Number of terms	Constant Term	Coefficient of 1st Term	Degree of 2nd Term	Degree of Polynomial	
$4x^2 + 6x - 9$						
$-17x^4y - 5y^3$						
$\left(\frac{1}{2}\right)w^6$						
$14 + 3m^4$						
$3x^3+4x^2 - x + 8$						
$-10mn + 4m^2n^2$						

Operations on Polynomials

Terms are **like** if they have exactly the same letter(s), and the exact exponents on each of the letters. Numbers without letters are like.

For example: $3x^2y$ and $8x^2y$ are **like**.

m^2n and mn are not **like**.

$7x$ and $5x$ are **like**.

-6 and $4w$ are not **like**.

Addition/Subtraction

To add or subtract terms, they must be **like**.

For example, $3x + 2x = 5x$. [think of the 'x' as an object. Three books plus two books is equal to 5 books]

For example, $3x + 2$ is equal to $3x + 2$. There is no combining or simplification because, $3x$ and 2 are not **like**.

For example, $7x^2 - 3x^2 = 4x^2$. [remember, if you think of the 'x^2' as an object, then 7 books take away 3 books is equal to 4 books]

For example, $7x^2 - 3x$ is equal to $7x^2 - 3x$. There is no combining or simplification because, $7x^2$ and $-3x$ are not **like**.

Practice

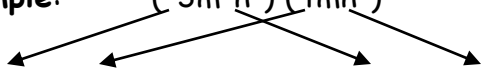
Add or subtract the following, if possible.

1. $6y + 9y - 2y$
2. $-4m^2n - m^2n$
3. $3w + 2 + 8w + 10$
4. $5v^3 + 7v + 5$
5. $18pxy + 6p - pxy$
6. $6 - 8c^2 + 4c - 2$
7. $-8m + 2m - 12m^4 + 1 - 3m^4$

Multiplication [Note: Terms do **not** have to be **like.**]

1. Monomials (multiplying 2 single terms together)

   For example: $(-3m^2n^3)(4mn^5)$

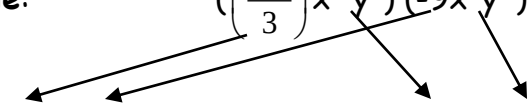
   

   Multiply coefficients: $(-3)(4)$

   Multiply powers with the same base: $(m^2)(m)(n^3)(n^5)$

   Final Answer: $-12m^3n^8$

   For example: $\left(\frac{-2}{3}\right)x^{-2}y^4)(-9x^7y^{-1})$

   Multiply coefficients: $\left(\frac{-2}{3}\right)(-9)$

   Multiply powers with the same base: $(x^{-2})(x^7)(y^4)(y^{-1})$

   Final Answer: $6x^5y^3$

2. Monomials by 2 or more terms (apply the above procedures with the distributive property).

   For Example: $2v(v - 8)$

   Using the distributive property, it means, $(2v)(v) - (2v)(8)$

   Final Answer: $2v^2 - 16v$

   For example: $-4wr^5(w + 2r^2 - 3w^3r^4)$

   Using the distributive property, it means,

   $(-4wr^5)(w) + (-4wr^5)(2r^2) - (-4wr^5)(3w^3r^4)$

   Final Answer: $-4w^2r^5 - 8wr^7 + 12w^4r^9$

3. Multiplying binomials

For example: $(x + 3)(x + 4)$

	x	+3
x		
+4		

Think of a field having one dimension as (x +3) and a second dimension as (x +4). Since we are multiplying length by width, we are finding the **area** of the field. When we find the area of each of the small 4 individual boxes, the total area is found by **adding** all the 4 pieces together.

	x	+3
x	x^2	3x
+4	4x	12

Final Answer: $x^2 + 7x + 12$

For example: $(2y^2 + 5y)(y - 8)$

	$2y^2$	5y
y	$2y^3$	$5y^2$
-8	$-16y^2$	-40y

Final Answer: $2y^3 - 11y^2 - 40y$

Practice

Multiply each of the following questions. Before starting the question, first identify the type of multiplication required (1. Monomial; 2. Monomial by 2 or more terms; 3. Binomials).

1. $5m(m + 10)$

   Type required: _____
   Answer: _____

2. $(-3v)(12v^6)$

   Type required: _____
   Answer: _____

3. $(x + 6)(x + 2)$

   Type required: _____
   Answer: _____

4. $\left[\left(\frac{-1}{3}\right)y^4w\right](12w)$

   Type required: _____
   Answer: _____

5. $-3xy(x + 5y - 8x^2y^3)$

   Type required: _____
   Answer: _____

6. $(2n - 3)(n - 1)$

   Type required: _____
   Answer: _____

7. $6(1 - 9x^3)$

   Type required: _____
   Answer: _____

8. $(2xy^2)(3xy)(-4x^3y^7)$

   Type required: _____
   Answer: _____

9. $(3x^3 + 7x)(-2x - 4)$

   Type required: _____
   Answer: _____

Division

1. Dividing Monomials

Divide coefficients $\frac{12x^5y^3}{-2x^2y}$ Divide powers with the same base

Final Answer $-6x^3y^2$

Simplify coefficients $\frac{-18mn^4}{-14m^2n}$ Divide powers with the same base

Final Answer $\frac{9n^3}{7m}$

2. Dividing 2 or more terms by a monomial.

$\frac{12x^4+6x^3}{2x}$ **means** $\frac{12x^4}{2x} + \frac{6x^3}{2x}$

Final Answer $6x^3 + 3x^2$

$\frac{-5w^5-10w+25w^3}{5w}$ **means** $\frac{-5w^5}{5w} - \frac{10w}{5w} + \frac{25w^3}{5w}$

Final Answer $-w^4 - 2 + 5w^2$

Remember: $\frac{4x+9}{2}$ **does not equal** $2x + 9$

It equals _____.

Practice

1. $\frac{6x^2+3}{3}$ = _____

2. $\frac{20m^5n^9}{-4mn^3}$ = _____

3. $\frac{-15y+3y^9-18y^2}{3y}$ = _____

4. $\frac{30r^3-15r^2}{10r}$ = _____

5. $(4x + 9) + (6x + 2)$ = _____

6. $17m^2 - 8 - 10 + m^2 + m$ = _____

7. $(-5r^5)(7r)$ = _____

8. $8v^3(v - 3v^2)$ = _____

9. $(x + 8)(x - 1)$ = _____

10. $3y(y - 1) + -4y(y + 5)$ _____

Polynomials(Solutions)

Polynomials are expressions having 1 or more terms.

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A

may have a **constant** term. A **constant** term is just a number without a letter.

Practice

Polynomial	Number of terms	Constant Term	Coefficient of 1st Term	Degree of 2nd Term	Degree of Polynomial	
$4x^2 + 6x - 9$	3	-9	4	1st	2nd	
$-17x^4y - 5y^3$	2	None	-17	3rd	4th	
$\left(\frac{1}{2}\right)w^6$	1	None	$\frac{1}{2}$	N/A	6th	
$14 + 3m^4$	2	14	14 (sometimes called a constant coefficient)	4th	4th	
$3x^3+4x^2 - x + 8$	4	8	3	2nd	3rd	

$-10mn + 4m^2n^2$	2	None	-10	4^{th}	4^{th}	

Operations on Polynomials

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For example: $3x^2y$ and $8x^2y$ are **like**.

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$7x$ and $5x$ are **like**.

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For example, $7x^2 - 3x$ is equal to $7x^2 - 3x$. There is no combining or simplification because, $7x^2$ and $-3x$ are not **like**.

Practice

Add or subtract the following, if possible.

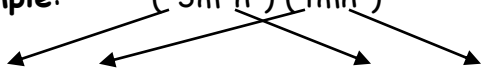
1. $6y + 9y - 2y$ = $13y$
2. $-4m^2n - m^2n$ = $-5m^2n$
3. $3w + 2 + 8w + 10$ = $11w + 12$

4. $5v^3 + 7v + 5$ = $5v^3 + 7v + 5$
5. $18pxy + 6p - pxy$ = $17pxy + 6p$
6. $6 - 8c^2 + 4c - 2$ = $-8c^2 + 4c + 4$
7. $-8m + 2m - 12m^4 + 1 - 3m^4$ = $-15m^4 - 6m + 1$

Multiplication [Note: Terms do **not** have to be **like.**]

1. Monomials (multiplying 2 single terms together)

For example: $(-3m^2n^3)(4mn^5)$

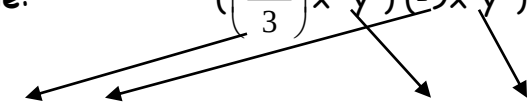


Multiply coefficients
$(-3)(4)$

Multiply powers with the same base
$(m^2)(m)(n^3)(n^5)$

Final Answer: $-12m^3n^8$

For example: $\left(\frac{-2}{3}\right)x^{-2}y^4)(-9x^7y^{-1})$



Multiply coefficients
$\left(\frac{-2}{3}\right)(-9)$

Multiply powers with the same base
$(x^{-2})(x^7)(y^4)(y^{-1})$

Final Answer: $6x^5y^3$

2. Monomials by 2 or more terms (apply the above procedures with the distributive property).

For example: $2v(v - 8)$

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Final Answer: $2v^2 - 16v$

For example: $-4wr^5(w + 2r^2 - 3w^3r^4)$

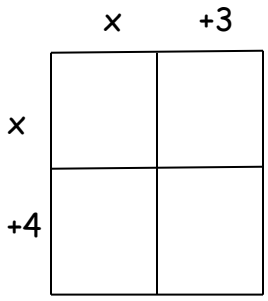
Using the distributive property, it means,

$(-4wr^5)(w) + (-4wr^5)(2r^2) - (-4wr^5)(3w^3r^4)$

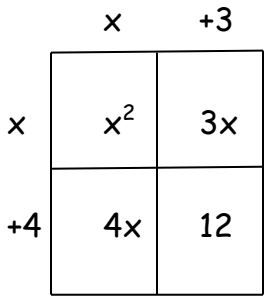
Final Answer: $-4w^2r^5 - 8wr^7 + 12w^4r^9$

3. Multiplying binomials

For example: $(x + 3)(x + 4)$



Think of a field having one dimension as (x +3) and a second dimension as (x +4). Since we are multiplying length by width, we are finding the **area** of the field. When we find the area of each of the small 4 individual boxes, the total area is found by **adding** all the 4 pieces together.



Final Answer: $x^2 + 7x + 12$

For example: $(2y^2 + 5y)(y - 8)$

	$2y^2$	$5y$
y	$2y^3$	$5y^2$
-8	$-16y^2$	$-40y$

Final Answer: $2y^3 - 11y^2 - 40y$

Practice

Multiply each of the following questions. Before starting the question, first identify the type of multiplication required (1. Monomial; 2. Monomial by 2 or more terms; 3. Binomials).

1. $5m(m + 10)$

   Type required: 2
   Answer: $5m^2+50m$

2. $(-3v)(12v^6)$

   Type required: 1
   Answer: $-36v^7$

3. $(x + 6)(x + 2)$

   Type required: 3
   Answer: $x^2+8x+12$

4. $\left[\left(\frac{-1}{3}\right)y^4w\right](12w)$

   Type required: 1
   Answer: $-4y^4w^2$

5. $-3xy(x + 5y - 8x^2y^3)$

   Type required: 2
   Answer: $-3x^2y-15xy^2+24x^3y^4$

6. $(2n - 3)(n - 1)$

   Type required: 3
   Answer: $2n^2-5n+3$

7. $6(1 - 9x^3)$

   Type required: 2
   Answer: $6-54x^3$

8. $(2xy^2)(3xy)(-4x^3y^7)$ Type required: ___1_____
Answer: $-24x^5y^{10}$

9. $(3x^3 + 7x)(-2x - 4)$ Type required: ___3_____
Answer: $-6x^4-12x^3-14x^2-28x$

Division

1. Dividing Monomials

Divide coefficients → $\frac{12x^5y^3}{-2x^2y}$ ← Divide powers with the same base

Final Answer $-6x^3y^2$

Simplify coefficients → $\frac{-18mn^4}{-14m^2n}$ ← Divide powers with the same base

Final Answer $\frac{9n^3}{7m}$

2. Dividing 2 or more terms by a monomial.

$\frac{12x^4+6x^3}{2x}$ **means** $\frac{12x^4}{2x} + \frac{6x^3}{2x}$

Final Answer $6x^3 + 3x^2$

$\frac{-5w^5-10w+25w^3}{5w}$ **means** $\frac{-5w^5}{5w} - \frac{10w}{5w} + \frac{25w^3}{5w}$

Final Answer $-w^4 - 2 + 5w^2$

Remember: $\frac{4x+9}{2}$ **does not equal** $2x + 9$

It equals __2x + 9/2__.

Practice

3. $\frac{6x^2+3}{3}$ = $2x^2+1$

4. $\frac{20m^5n^9}{-4mn^3}$ = $-5m^4n^6$

5. $\frac{-15y+3y^9-18y^2}{3y}$ = $-5+y^8-6y$

6. $\frac{30r^3-15r^2}{10r}$ = $3r^2-1.5r$

7. $(4x + 9) + (6x + 2) =$ $10x+11$

8. $17m^2 - 8 - 10 + m^2 + m =$ $18m^2+m-18$

9. $(-5r^5)(7r) =$ $-35r^6$

10. $8v^3(v - 3v^2) =$ $8v^4-24v^5$

11. $(x + 8)(x - 1) =$ x^2+7x-8

12. $3y(y - 1) + -4y(y + 5)$

Perform the multiplication first, to clear the brackets.

$3y^2 - 3y + (-4y^2 - 20y)$

$3y^2 - 3y - 4y^2 - 20y$

Combine like terms.

$-y^2 - 23y$