Permutations Part 1 Solutions

1. Find the number of pathways from A to C, but having to pass through B.

Begin by calculating the number of ways to move from A to B. There are 2 blocks east and 1 block north. Think of this as arranging a 3-letter 'word' with repetitions. We would be arranging the letters EEN. This would be $\frac{3!}{2!}$, which is equal to 3.

From B to C, the movement can be stated as 3 blocks east and 1 block north. Think of this as arranging a 4-letter 'word' with repetitions. We would be arranging the letters EEEN. This would be $\frac{4!}{3!}$, which is equal to 4.

Think of the total process consisting of 2 stages; from A to B, and then from B to C. Using the Fundamental Counting Principle, we find the choices for each stage and then multiply these numbers together. In this case, it will be (3)(4).

There are 12 possible pathways from A to C, passing through B.

2. Refer to the pathway diagram above. The events, 'passing through B' and 'not passing through B', are complementary. This means that the pathways from A to C (always moving up or right) either go through B or they do not. There are no other options. The pathways that do not pass through B is equal to the total number of pathways from A to C subtract the number that pass through B. How many pathways do not pass through B?

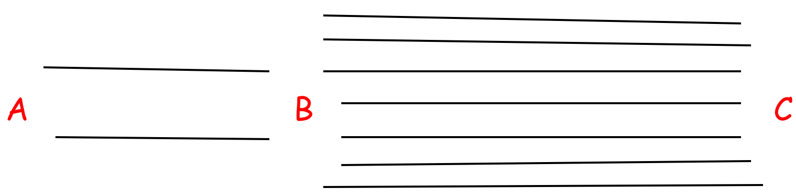
The movement from A to C can be described as going 5 block east and 2 blocks north. We would be arranging the letters, EEEEENN. This would be $\frac{7!}{5!2!}$, which is equal to $\frac{5040}{240}$, which is equal to 21.

Subtracting the number of ways passing through B (12) from the total number of ways from A to C (21), will result in the number of ways of going from A to C and not passing through B.

The number of pathways that do not pass through B is 21 – 12 or 9.

3. There are 2 roads between cities A and B. There are 7 roads between cities B and C.

   a) How many ways can a person travel from A to C by way of B?



(2)(7) = 14

There are 14 ways to travel from City A to City C, by way of B.

   b) How many ways can a person make a round trip from A to C and back to A by way of B, without using the same road twice?

On the trip from A to C, we have already calculated that there are 14 ways.

On the return trip, from C to B, there are no only 6 choices, because the road taken to C is not part of our selection [without using the same road twice]. As well, on the return trip from B to A, there is only 1 choice [without using the same road twice]. The number of ways for the return trip is (6)(1) or 6.

Using the Fundamental Counting Principle, we have 2 stages; travelling to C, and then the return trip. The total number of choices for each stage is 14 (there) and 6 (return). Multiplying these numbers together, will produce the answer.

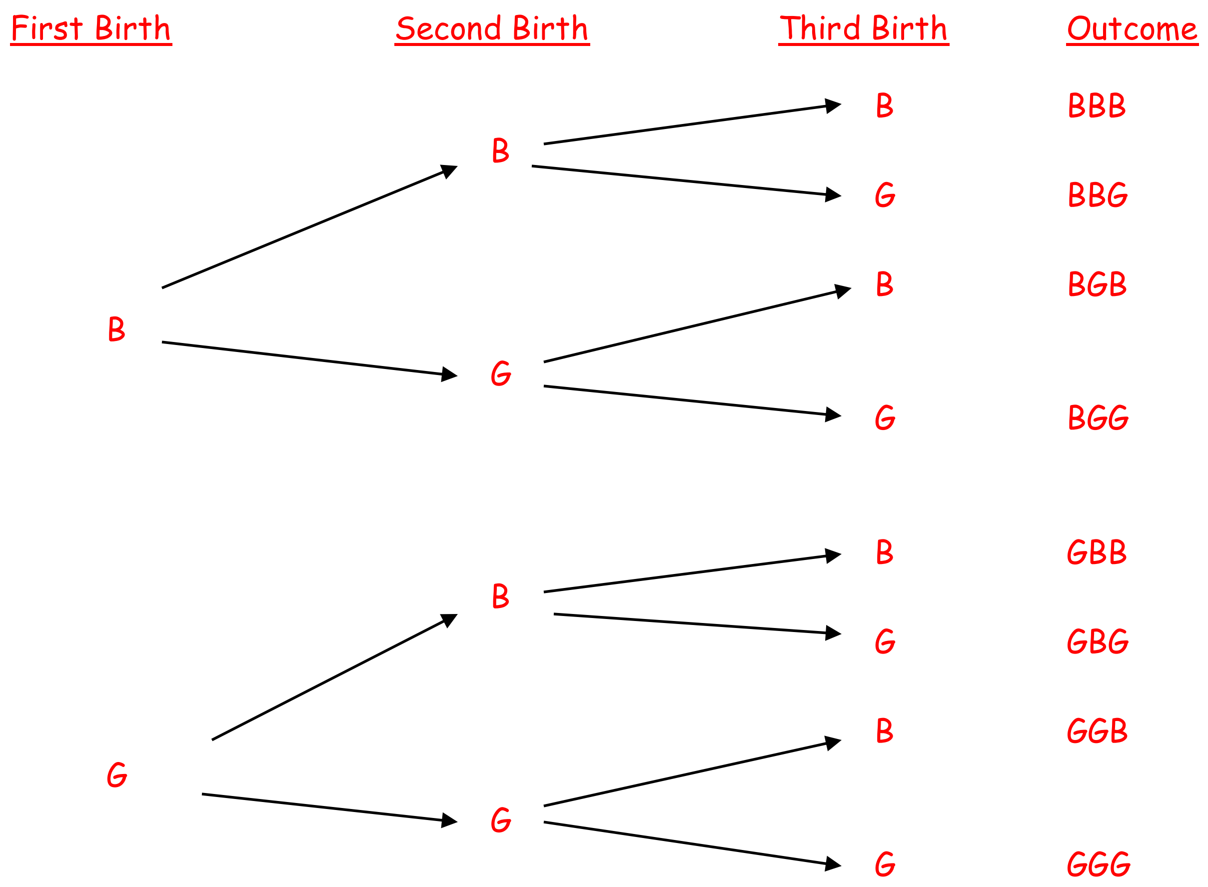
The number of ways a person can make a round trip from A to C and back to A by way of B, without using the same road twice, is (14)(6) or 84.

4. Suppose a person has a choice of 6 different computers, 3 different monitors and 5 different printers. How many ways can he/she select a computer system?

Using the Fundamental Counting Principle, three stages are identified; computers, monitors and printers. Taking the number of choices for each stage and multiplying them together, we get (6)(3)(5), which is equal to 90.

There are 90 ways he/she can select a computer system.

5. Show a tree diagram to list all the possible outcomes for the gender of the children in a family that has 3 children.



6. Determine the total number of licence plates if each plate consists of 3 letters followed by 3 numbers.

There are 6 stages:

____	____	____	____	____	____
Letters	Letters	Letters	Numbers	Numbers	Numbers

The number of choices for each stage:

_26_	_26_	_26_	_10_	_10_	_10_
Letters	Letters	Letters	Numbers	Numbers	Numbers

Using the Fundamental Counting Principle, multiply the choices together.

26 X 26 X 26 X 10 X 10 X 10 = 17 576 000

The total number of licence plates is 17 576 000.

7. Simplify each expression.

a) $\frac{n!}{n}$ b) $\frac{(2)(n!)}{(4)(n-1)(n-3)!}$ c) $\frac{(n-1)!(n+1)!}{(n!)^2}$

a) $\frac{n(n-1)!}{n}$

$= (n - 1)!$

b) $\frac{n(n-1)(n-2)(n-3)!}{2(n-1)(n-3)!}$

$= \frac{n(n-2)}{2}$

c) $\frac{(n-1)!(n+1)n!}{n!n!}$

$$= \frac{(n-1)!(n+1)}{n(n-1)!}$$

$$= \frac{n+1}{n}$$

8. Solve for n, given $(2)({}_7P_2) = {}_nP_2 + 12$

Convert any expressions to their natural number equivalent. (${}_7P_2 = 42$). Multiplying (42) by (2), and then subtracting 12 from both sides of the equal sign, we get

$72 = {}_nP_2$

Rewrite the right side of the equal sign with the equivalent form from the formula sheet.

$$72 = \frac{n!}{(n-2)!}$$

$$72 = \frac{n(n-1)(n-2)!}{(n-2)!}$$

$72 = n(n - 1)$

$0 = n^2 - n - 72$

Factor.

$0 = (n - 9)(n + 8)$

Using the zero principle,

$n = 9$

The value of n is 9.

9. Given ${}_nP_2 = 132$, the value of n must be a natural number. In the process of solving for n, what is the extraneous root?

Convert ${}_nP_2$ to its formula equivalent.

$$\frac{n!}{(n-2)!} = 132$$

$$\frac{n(n-1)(n-2)!}{(n-2)!} = 132$$

$n(n-1) = 132$

$n^2 - n - 132 = 0$

$(n - 12)(n + 11) = 0$

$n = 12$ or $n = -11$

The extraneous root is -11.

10. A football team plays a 12 game schedule. How many ways can the schedule end with 7 wins, 3 losses and 2 ties?

If we let Wins = W

Losses = L

Ties = T,

The 12 game schedule can be viewed as arranging 12 letters,

WWWWWWWLLLTT, with repetitions.

$\frac{12!}{7!3!2!}$ = 7920.

There are 7920 ways the schedule can end with 7 wins, 3 losses and 2 ties.

11. How many different 10-letter 'words' be made using the letters from the word STATISTICS?

$\frac{10!}{3!3!2!}$ = 50 400

There are 50 400 'words' that can be made using the letters from the word statistics.

12. There are 12 available seats on an aircraft. If 7 customers are each booking 1 seat, how many different ways could they be assigned a seat?

Think of 7 customers.

___ ___ ___ ___ ___ ___ ___

The first customer has a choice of 12 available seats. The second customer now has a choice of only 11 available seats. The third customer now has only a choice of 10 seats, and so on.

12 X 11 X 10 X 9 X 8 X 7 X 6 = 3 991 680 [This is the same as $_{12}P_7$]

There are 3 991 680 ways to assign the seats.

13. A school has 10 doors. How many ways can you enter and leave the school if you cannot leave by the same door you entered?

There are two stages; entering and exiting. There are 10 choices to enter, and 9 choices to exit. Using the Fundamental Counting Principle, multiply (10)(9).

There are 90 ways to enter the school, if you have to exit by a different door.

14. The score at the end of the second period of a hockey game is 3-2 for the home team. How many different scores were possible at the end of the first period?

Think of each team being a stage.

The home team could have a score of 0, 1, 2, or 3 at the end of the first period.[In other words, there are 4 choices for this stage]

The visiting team could have a score of 1,2 or 2 at the end of the first period.[In other words, there are 3 choices for this stage]

Multiply (4)(3) to get 12.

There are 12 different possible scores.