Isolating a Variable Practice

1. The formula for the surface area of a cylinder is SA = $2\pi r^2 + 2\pi rh$. When h is isolated, the equivalent equation is

   A) $\frac{SA + 2\pi r^2}{2\pi r}$ B) $\frac{SA - 2\pi r^2}{2\pi r}$ C) $\frac{SA + 2\pi r}{2\pi r^2}$ D) $\frac{SA - 2\pi r}{2\pi r^2}$

Use the following information to answer the next two questions.

In 1805, Rear-Admiral Beaufort created a numerical scale to help sailors quickly assess the strength of the wind. The integer scale ranges from 0 to 12. The wind scale, B, is related to the wind velocity, v, in km/hr, by the formula
$B = 1.33\sqrt{v + 10.0} - 3.49,\ v \geq -10.$

John was asked to isolate v. His work is shown below.

Step 1	$B + 3.49 = 1.33\sqrt{v + 10.0} - 3.49 + 3.49$
Step 2	$B + 3.49 - 1.33 = 1.33\sqrt{v + 10.0} - 1.33$
Step 3	$B + 3.49 - 1.33 = 1.33\sqrt{v + 10.0} - 1.33$
Step 4	$B + 2.16 = \sqrt{v + 10.0}$
Step 5	$(B + 2.16)^2 = \left(\sqrt{v + 10.0}\right)^2$
Step 6	$(B + 2.16)^2 - 10.0 = v$

2. Unfortunately, John's work is incorrect. His first error was made in step

   A) 1 B) 2 C) 3 D) 4

3. Explain his error and state what he should have done.

Use the following information to answer the next question.

The path of a particular fireworks rocket is modelled by the function $h(t) = -4.9(t - 3)^2 + 47$, where h is the rocket's height above the water, in metres, at time, t.

4. Isolate t in terms of h(t) and explain your steps

5. The correct isolation of *y* in the equation, $\frac{29m}{0.5+y} = 8$ is

A) $y = \frac{29m-4}{8}$ B) $y = \frac{29m}{8} - 4$ C) $y = \frac{25m}{8}$ D) $y = \frac{58m+4}{8}$

6. Isolate *w* given the equation, $4x = \sqrt{\frac{w+9}{2}}$

7. Given, $d = \frac{1}{2}at^2$, the correct isolation of *t*, is

A) $t = \frac{2d}{a}$ B) $t = \frac{2a}{d}$ C) $t = \pm\sqrt{\frac{2d}{a}}$ D) $t = \frac{\pm\sqrt{2d}}{a}$

8. Solve the formula, $V = \frac{1}{3}\pi h^2(3r - h)$, for r.

9. Isolate N, in the equation, $I = \frac{PN}{RN + A}$

Isolating a Variable Practice Solutions

1. The formula for the surface area of a cylinder is $SA = 2\pi r^2 + 2\pi rh$. When h is isolated, the equivalent equation is

   A) $\frac{SA + 2\pi r^2}{2\pi r}$ B) $\frac{SA - 2\pi r^2}{2\pi r}$ C) $\frac{SA + 2\pi r}{2\pi r^2}$ D) $\frac{SA - 2\pi r}{2\pi r^2}$

Solution

$SA = 2\pi r^2 + 2\pi rh$

Our first step is to isolate the term containing 'h', which is the second term on the right side of the equal sign. Therefore, subtract $2\pi r^2$ from both sides of the equal sign.

$SA - 2\pi r^2 = 2\pi r^2 - 2\pi r^2 + 2\pi rh$

$SA - 2\pi r^2 = 2\pi rh$

The variable we want to isolate is 'h', and it is being *multiplied* by $2\pi r$, so the next step is to *divide* both sides of the equal sign by $2\pi r$.

The final answer is B, $\frac{SA - 2\pi r^2}{2\pi r}$

Use the following information to answer the next two questions.

In 1805, Rear-Admiral Beaufort created a numerical scale to help sailors quickly assess the strength of the wind. The integer scale ranges from 0 to 12. The wind scale, B, is related to the wind velocity, v, in km/hr, by the formula

$B = 1.33\sqrt{v + 10.0} - 3.49,\ v \geq -10.$

John was asked to isolate v. His work is shown below.

Step 1	$B + 3.49 = 1.33\sqrt{v+10.0} - 3.49 + 3.49$
Step 2	$B + 3.49 - 1.33 = 1.33\sqrt{v+10.0} - 1.33$
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Step 5	$(B + 2.16)^2 = \left(\sqrt{v+10.0}\right)^2$
Step 6	$(B + 2.16)^2 - 10.0 = v$

2. Unfortunately, John's work is incorrect. His first error was made in step

   A) 1 B) 2 C) 3 D) 4

3. Explain his error and state what he should have done.

Solution

In step 2, John subtracted 1.33 from both sides of the equal sign. Since 1.33 is *multiplied* by the radical containing the variable we want, 1.33 should have been *divided* on both sides.

Use the following information to answer the next question.

The path of a particular fireworks rocket is modelled by the function $h(t) = -4.9(t - 3)^2 + 47$, where h is the rocket's height above the water, in metres, at time, t.

4. Isolate t in terms of h(t) and explain your steps

Solution

Isolating t, in terms of h(t), means that h(t) will be included in the answer.

Think of $-4.9(t - 3)^2$ as a single term. The term 47 is not wanted and needs to be moved.

Subtract 47 from both sides.

$h(t) - 47 = -4.9(t - 3)^2 + 47 - 47$

$h(t) - 47 = -4.9(t - 3)^2$

Divide both sides by -4.9

$$\frac{h(t)-47}{-4.9} = \frac{-4.9(t-3)^2}{-4.9}$$

$$\frac{h(t)-47}{-4.9} = (t-3)^2$$

Take the square root of both sides.

$$\pm\sqrt{\frac{h(t)-47}{-4.9}} = \sqrt{(t-3)^2}$$

The square root of anything squared is itself.

$$\pm\sqrt{\frac{h(t)-47}{-4.9}} = t-3$$

Add 3 to both sides.

$$\pm\sqrt{\frac{h(t)-47}{-4.9}}+3=t$$

5. The correct isolation of y in the equation, $\frac{29m}{0.5+y}=8$ is

A) $y=\frac{29m-4}{8}$ ← Ans B) $y=\frac{29m}{8}-4$ C) $y=\frac{25m}{8}$ D) $y=\frac{58m+4}{8}$

Solution

Clear the fraction by multiplying both sides by 0.5 + y.

$$(0.5+y)\left(\frac{29m}{0.5+y}=8\right)$$

29m = 8(0.5 + y)

29m = 4 + 8y

Subtract 4 from both sides.

29m – 4 = 8y

Divide every term by 8.

$$y=\frac{29m-4}{8}$$

The correct answer is A.

6. Isolate *w* given the equation, $4x = \sqrt{\frac{w+9}{2}}$

Solution

Square both sides to remove the radical sign. *[Remember that multiplying a radical by itself, results in just the radicand, and removes the radical sign. For example,* $\sqrt{5} X \sqrt{5} = 5$ *]*

$$(4x)^2 = \left(\sqrt{\frac{w+9}{2}}\right)^2$$

$$16x^2 = \frac{w+9}{2}$$

Clear the fraction by multiplying every term by 2.

$32x^2 = w + 9$

Subtract 9 from both sides.

$32x^2 - 9 = w$

or

$w = 32x^2 - 9$

7. Given, $d = \frac{1}{2}at^2$, the correct isolation of *t*, is

A) $t = \frac{2d}{a}$ B) $t = \frac{2a}{d}$ C) $t = \pm\sqrt{\frac{2d}{a}}$ D) $t = \frac{\pm\sqrt{2d}}{a}$

Solution

Multiply every term by 2 to clear the fraction.

$2d = at^2$

Divide both sides by 'a'.

$$\frac{2d}{a} = \frac{at^2}{a}$$

= $$\frac{2d}{a} = t^2$$

Take the square root of both sides of the equal sign.

$$t = \pm\sqrt{\frac{2d}{a}}$$

The correct answer is C.

8. Solve the formula, $V = \frac{1}{3}\pi h^2(3r - h)$, for r.

Solution

The variable we want, r, is in brackets. The first step is to isolate the brackets. Since $\frac{1}{3}\pi h^2$ is *multiplied* by (3r – h), *divide* both sides by $\frac{1}{3}\pi h^2$.

$$\frac{V}{\frac{1}{3}\pi h^2} = \frac{\frac{1}{3}\pi h^2(3r - h)}{\frac{1}{3}\pi h^2}$$

= $$\frac{V}{\frac{1}{3}\pi h^2} = (3r - h)$$

Add h to both sides.

$$\frac{V}{\frac{1}{3}\pi h^2} + h = 3r$$

Divide every term by 3.

$$\frac{\frac{V}{\frac{1}{3}\pi h^2}}{3} + \frac{h}{3} = \frac{3r}{3}$$

$$\frac{V}{\pi h^2} + \frac{h}{3} = r$$

Alternatively,

Begin by multiplying each term by 3 to clear the fraction.

$3V = \pi h^2(3r - h)$

Divide both sides by πh^2

$$\frac{3V}{\pi h^2} = 3r - h$$

Add h to both sides.

$$\frac{3V}{\pi h^2} + h = 3r$$

Divide every term by 3.

$$\frac{V}{\pi h^2} + \frac{h}{3} = r$$

9. Isolate N, in the equation, $I = \frac{PN}{RN + A}$

Solution

Cross multiply to eliminate the fraction.

(I)(RN + A) = PN

Multiply to remove the brackets.

IRN + IA = PN

Gather all 'N' terms to one side, and all non 'N' terms to the other side.

IRN – PN = -IA

Factor.

N(IR – P) = -IA

Divide both sides by (IR – P).

$$N = \frac{-IA}{IR - P}$$