Inverse of a Relation Solutions

1. If point A(2,-5) is on y = f(x), what is the image point A′ on the inverse of y = f(x)?

When points are moved because of a reflection in the line y = x (inverse), the x and y coordinates are interchanged.

The image point A′ is (-5,2).

2. Given the function f(x) = 6x + 1, determine $f^{-1}(7)$.

The notation $f^{-1}(7)$ means to find the value of the inverse when x = 7. If x = 7 for the inverse, y = 7 for f(x).

7 = 6x + 1

6 = 6x

1 = x

$f^{-1}(7) = 1$

3. If the point (m,n) is on y = f(x), the image point on $y = f^{-1}(x) + 2$ is (n, m+2).

   If the point (w,v) is on y = f(x), what is the image point on $y = f^{-1}(x) - 5$?

(v, w – 5)

4. For each of the following equations, write the equation of the inverse.

a) $y = -9x + 2$ b) $y + 3 = \frac{1}{4}(x - 1)^2$

To determine the equation of the inverse, interchange the x and the y and solve the equation for y.

a) $y = -9x + 2$

$x = -9y + 2$

$x - 2 = -9y$

$\frac{x-2}{-9} = y$ or

$y = \frac{-x+2}{9}$

b) $y + 3 = \frac{1}{4}(x - 1)^2$

$x + 3 = \frac{1}{4}(y - 1)^2$

$4x + 12 = (y - 1)^2$

$\pm\sqrt{4x+12} = \sqrt{(y-1)^2}$

$\pm\sqrt{4x+12} = y - 1$

$y = \pm\sqrt{4x+12} + 1$

5. What is an example of a restriction that can be placed on the function $y = x^2 - 10$, such that its inverse is a function?

The function above is quadratic, composed of 2 separate branches, one on each side of the vertex. As long as a horizontal line can be drawn on the original function which intersects the graph at more than one point, the inverse is not a function.

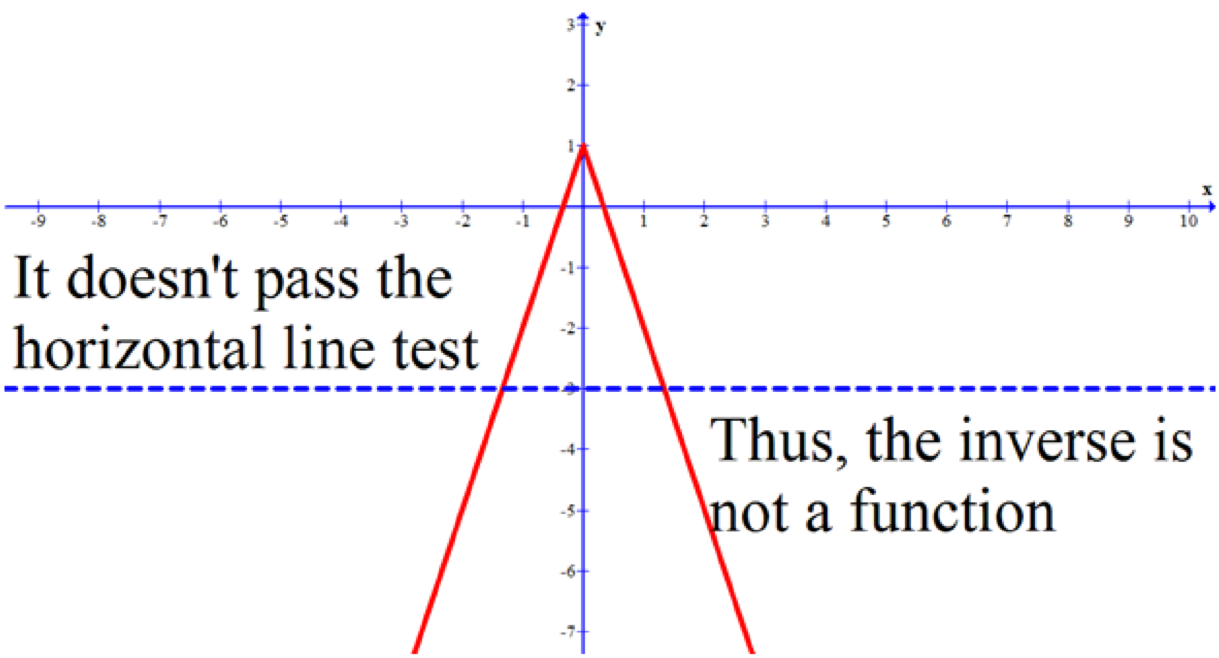
If the domain can be restricted such that only 1 (or only a part of 1) of the 2 branches is drawn, the inverse would then be a function.

There are multiple answers to this question; but the most common is to restrict the domain on either side of the vertex. This would create only a single branch.

In this case, restricting the domain to be either **$x \geq 0$, or $x \leq 0$** will work.

6. Will the inverse of the function, $y = -3|x| + 1$, also be a function? How do you know?

The graph below is $y = -3|x| + 1$. Since it doesn't pass the horizontal line test, its inverse is not a function.



7. Determine the invariant point of f(x) = 3x + 6 and its inverse.

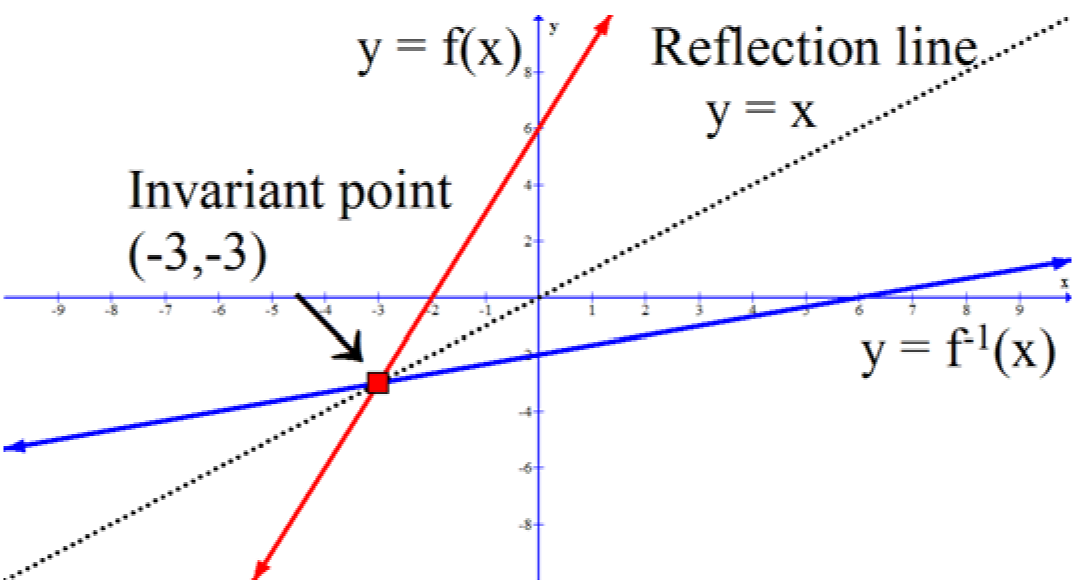
First find the equation of the inverse.

y = 3x + 6

x = 3y + 6

y = $\frac{x-6}{3}$

Graph the 2 equations.

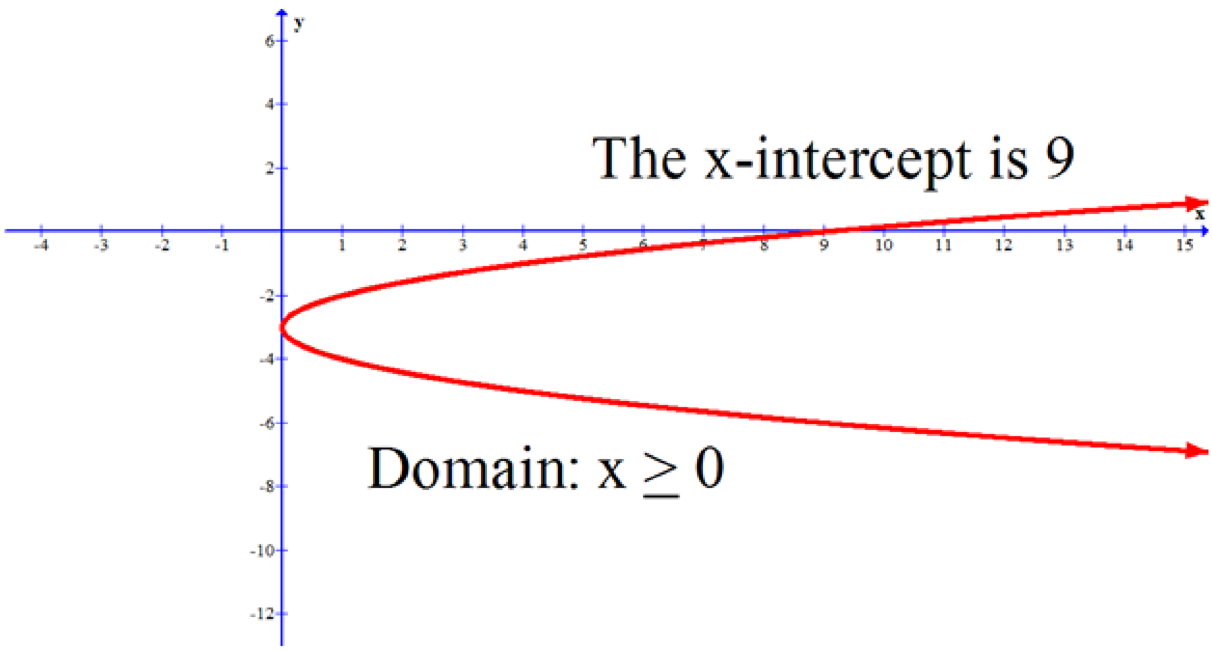


8. Given y = $(x + 3)^2$, what is the domain and the x-intercept of its inverse?

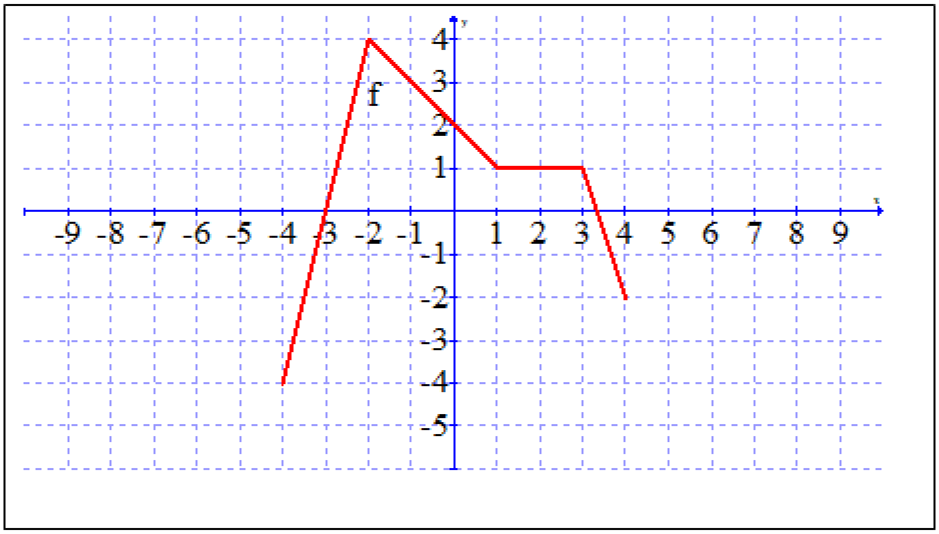
The equation of the inverse is:

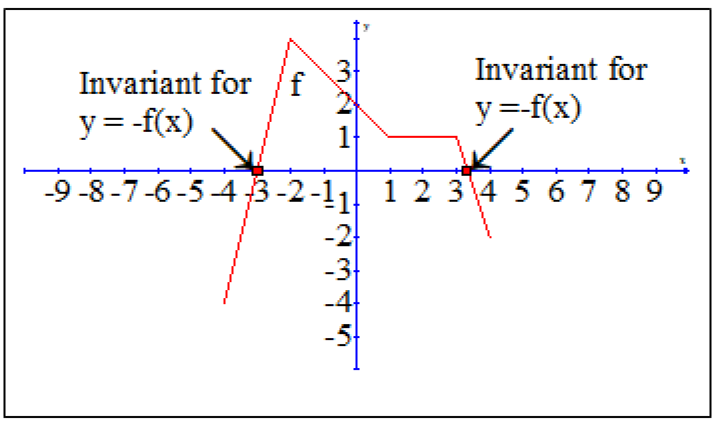
y = $\pm\sqrt{x} - 3$

The graph is shown below.



Use the following graph to answer the next question.

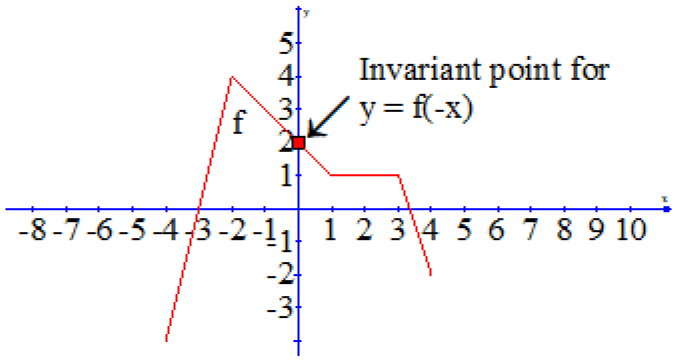




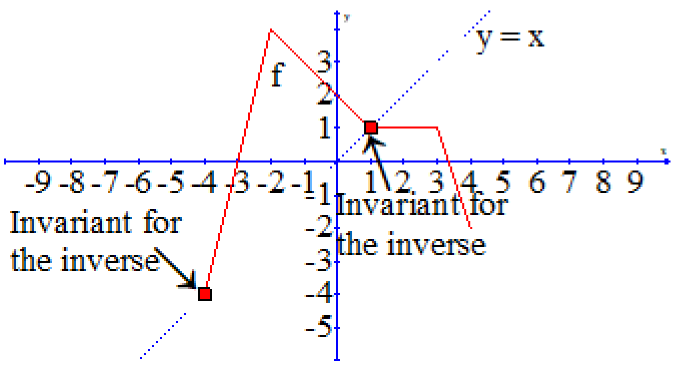
9. Consider 3 reflections on the graph of y =f(x) above. How many invariant points exist on

   a) y = -f(x) b) y = f(-x) c) the inverse of f(x)

a) For y = -f(x), there are 2 invariant points.



b) For y = f(-x), there is 1 invariant point.



c) For the inverse, there are 2 invariant points.

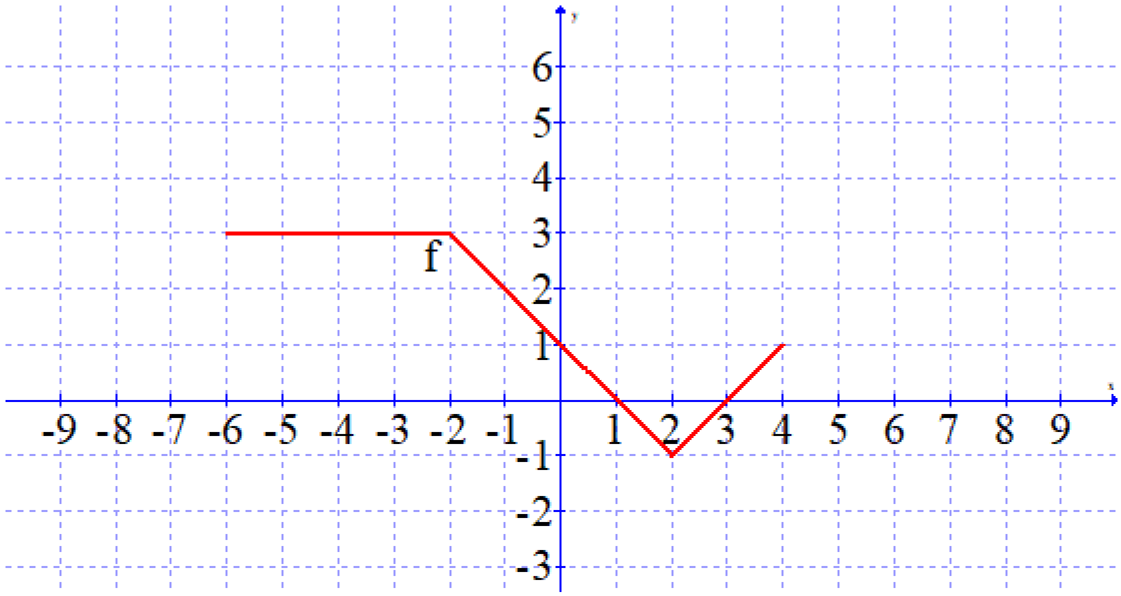
10. If the number of invariant points on y = -f(x) is m, the number of invariant points reflected in the line x = 0 is n, and the number of invariant points reflected in the line y = x is k, which statement below is correct?

The number of invariant points on y = -f(x) is 2. Thus, m = 2.

The number of invariant points on y = f(-x) is 1. Thus, n = 1.

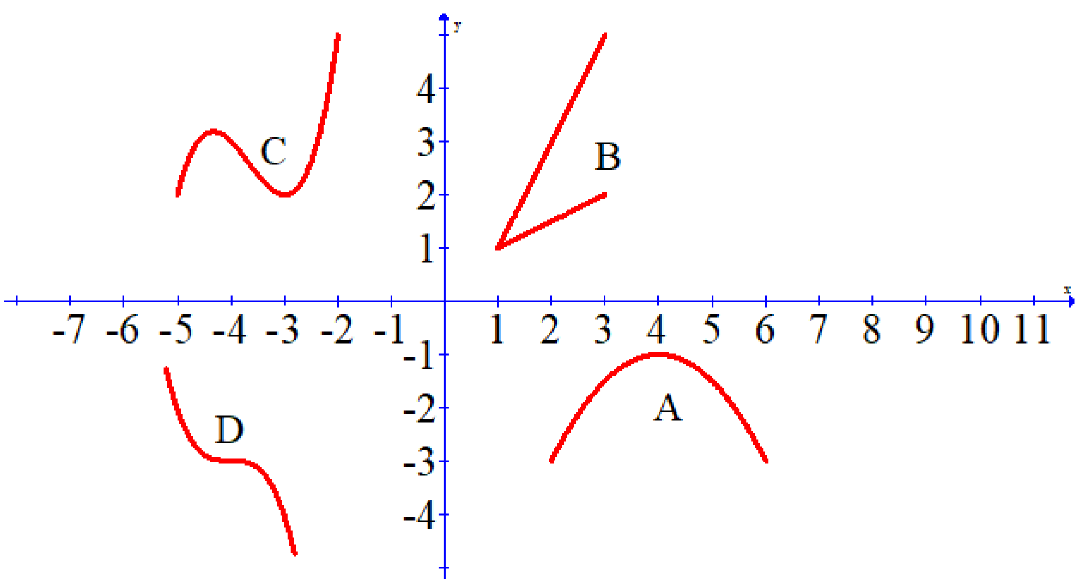
The number of invariant points on the inverse is 1. Thus, k = 1.

Statement c is correct because n + k = m [1 + 1 = 2]



a) m = k　　b) m + n = k　　c) n + k = m　　d) m = n

Use the graph below to answer the next question.



11. On the grid above, there are 4 separate partial graphs, 1 drawn in each of the 4 quadrants. If the graph of each inverse were drawn, and if no additional restrictions are given, which graph will have an inverse that is a function?

The correct answer is D, because D is the only graph to pass the horizontal line test.

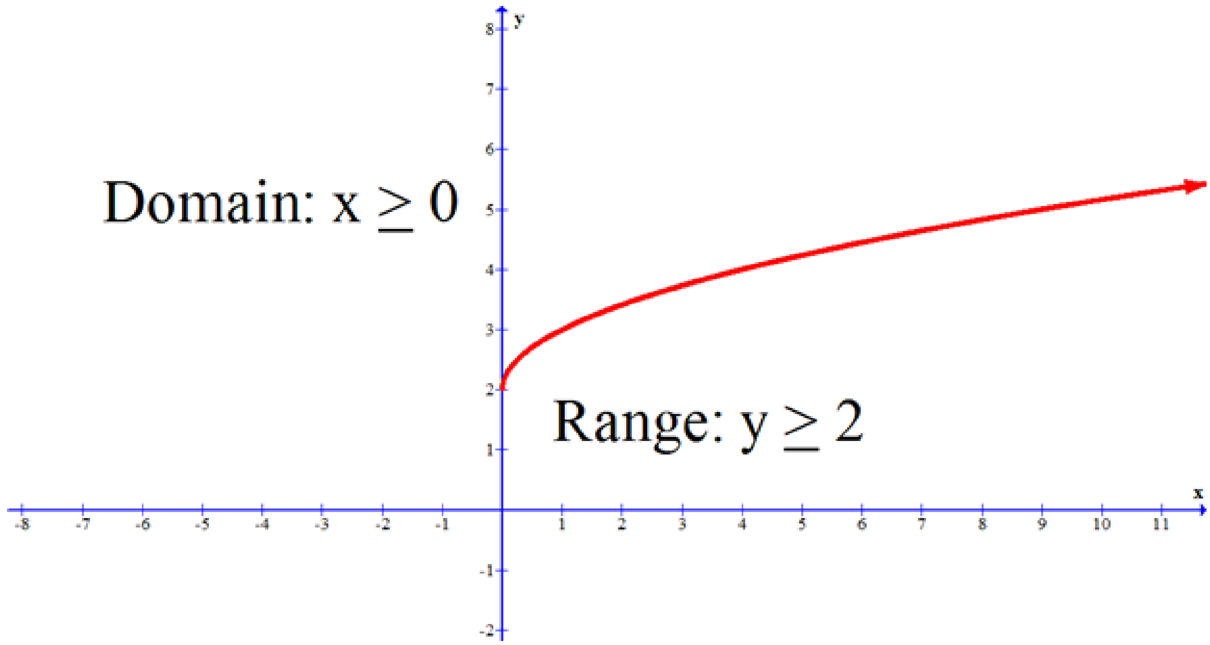
12. a) Algebraically determine the inverse of $f(x) = (x - 2)^2$, where $x > 2$.

$y = (x - 2)^2$

$x = (y - 2)^2$

$\sqrt{x} + 2 = y$

b) State the domain and range of $f^{-1}(x)$.



c)If (2,3) is on f(x), where does it move given $y = f^{-1}(x - 1)$?

If (2,3) is on f(x), then (3,2) is on $f^{-1}(x)$.

Now move (3,2) given that there is a horizontal translation 1 unit right.

The mapping notation is:

$(3,2) \rightarrow (3 + 1, 2)$

The point (3,2) moves to **(4, 2)**