Fractions

Equivalent Fractions

Equivalent fractions are derived when the numerator and denominator are either **multiplied** or **divided** by the same number.

For example: $$\frac{1}{2} = \frac{2}{4} = \frac{6}{12} = \frac{24}{48} = \frac{50}{100} = \frac{100}{200}$$
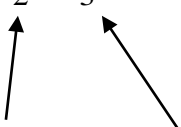
For example: $$\frac{64}{96} = \frac{32}{48} = \frac{16}{24} = \frac{8}{12} = \frac{4}{6} = \frac{2}{3}$$

Fractions are often written in simplest form. For the 6 equivalent fractions listed in example 1, $\frac{1}{2}$ is the fraction in simplest form. For the 6 equivalent fractions listed in example 2, $\frac{2}{3}$ is the fraction in simplest form.

Adding and Subtracting Fractions

In order to add or subtract fractions, the denominator must be the same. If denominators are not the same, we can use the concept of equivalent fractions to make them the same.

For example, given the question, $\frac{1}{2} + \frac{2}{3}$, the most common **error** would likely result in an answer of $\frac{3}{5}$.

The question above is adding 'halves' and 'thirds'. Since we need the denominator to be the same, we need to re-write as equivalent fractions. In this case, the lowest common denominator (LCD) is 6, because that is the lowest number that 2 and 3 divide evenly into. So, we need to express $\frac{1}{2}$ in 'sixths' and express $\frac{2}{3}$ in 'sixths'.

$\dfrac{1}{2} = \dfrac{x}{6}$;   since we multiply the 2 by **3** to get 6 on the bottom, multiply the 1 on the top by **3**, to get 3 on the top.

$\dfrac{1}{2} = \dfrac{3}{6}$.   Thus, we have expressed $\dfrac{1}{2}$ in 'sixths'.

$\dfrac{2}{3} = \dfrac{x}{6}$;   since we multiply the 3 by **2** to get 6 on the bottom, multiply the 2 on the top by **2** to get 4.

$\dfrac{2}{3} = \dfrac{4}{6}$ Thus, we have expressed $\dfrac{2}{3}$ in 'sixths'.

Now, $\dfrac{3}{6} + \dfrac{4}{6} = \dfrac{7}{6}$ **[Add numerator and keep denominator]**

For example, given the question, $\dfrac{3}{5} - \dfrac{1}{4}$, the most common **error** would likely result in an answer of $\dfrac{2}{1}$

The question above is subtracting 'fourths' from 'fifths'. Since we need the denominator to be the same, we need to re-write as equivalent fractions. In this case, the lowest common denominator (LCD) is 20 because that is the smallest number that 5 and 4 divide evenly into. So we need to express $\dfrac{3}{5}$ in 'twentyiths' and express $\dfrac{1}{4}$ in 'twentyiths'.

$\dfrac{3}{5} = \dfrac{x}{20}$ since we multiply the 5 by **4** to get 20 on the bottom, we multiply the 3 by **4** to get 12 on the top.

$\dfrac{3}{5} = \dfrac{12}{20}$ Thus we have expressed $\dfrac{3}{5}$ in 'twentyiths.'

$\dfrac{1}{4} = \dfrac{x}{20}$ since we multiply the 4 by **5** to get the 20 on the bottom, we multiply

the 1 by **5** to get 5 on the top.

$\dfrac{1}{4} = \dfrac{5}{20}$ Thus we have expressed $\dfrac{1}{4}$ in 'twentyiths'.

Now, $\dfrac{12}{20} - \dfrac{5}{20} = \dfrac{7}{20}$ **[Subtract numerators and keep denominator]**

Multiplying Fractions

Common denominators are not required when multiplying. Answers are calculated by multiplying numerators together, and then multiplying denominators together.

For example: $\left(\dfrac{3}{4}\right)\left(\dfrac{2}{3}\right) = \left(\dfrac{6}{12}\right)$

which is equal to $\left(\dfrac{1}{2}\right)$ in simplest form.

For example: $(5)\left(\dfrac{1}{2}\right)$ is the same as $\left(\dfrac{5}{1}\right)\left(\dfrac{1}{2}\right) = \left(\dfrac{5}{2}\right)$

For example: $\left(3\dfrac{1}{3}\right)\left(\dfrac{5}{4}\right)$ is the same as $\left(\dfrac{10}{3}\right)\left(\dfrac{5}{4}\right) = \left(\dfrac{50}{12}\right)$

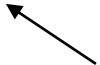which is equal to $\left(\dfrac{25}{6}\right)$ in simplest form.

Dividing Fractions

When dividing fractions, we multiply by the **reciprocal** of the **divisor**.

The **reciprocal** of a number is determined by interchanging the numerator with the denominator.

For example: The **reciprocal** of $\dfrac{3}{5}$ is $\dfrac{5}{3}$

The **divisor** is the number that follows the division sign.

For example: Given, $\dfrac{4}{7} \div \dfrac{2}{9}$

$\dfrac{2}{9}$ is the **divisor**

For example: $\dfrac{1}{2} \div \dfrac{3}{7}$ is the same as

$\left(\dfrac{1}{2}\right)\left(\dfrac{7}{3}\right)$ which is equal to $\left(\dfrac{7}{6}\right)$

For example: $\dfrac{3}{4} \div 6$ is the same as

$\dfrac{3}{4} \div \dfrac{6}{1}$ which is the same as

$\left(\dfrac{3}{4}\right)\left(\dfrac{1}{6}\right)$ which is equal to $\left(\dfrac{3}{24}\right)$

which is equal to $\left(\dfrac{1}{8}\right)$ in simplest form.

For example: $8 \div \left(2\dfrac{1}{2}\right)$ is the same as

$\left(\dfrac{8}{1}\right) \div \left(\dfrac{5}{2}\right)$ which is the same as

$$\left(\frac{8}{1}\right)\left(\frac{2}{5}\right) = \left(\frac{16}{5}\right)$$

Practice

1. $\frac{2}{3} + \frac{1}{3} =$ _____

2. $\frac{3}{4} + \frac{1}{6} =$ _____

3. $\frac{11}{12} - \frac{6}{12} =$ _____

4. $\frac{4}{5} - \frac{2}{3} =$ _____

5. $\left(\frac{1}{2} + \frac{1}{3}\right) - \frac{5}{12}$ _____

6. $\left(\frac{3}{4} - \frac{1}{5}\right) + \frac{3}{10}$ _____

7. $\frac{2}{9} + \frac{1}{3} =$ _____

8. $\frac{5}{6} - \frac{2}{3} =$ _____

9. $10 - \left(2\frac{1}{5}\right) =$ _____

10. $\left(3\frac{1}{2}\right) + 4 =$ _____

11. $\left(\dfrac{1}{11}\right)\left(\dfrac{1}{2}\right) =$ _____

12. $\left(\dfrac{3}{7}\right)\left(\dfrac{2}{3}\right) =$ _____

13. $4\left(\dfrac{1}{4}\right) =$ _____

14. $\left(\dfrac{3}{8}\right)(9) =$ _____

15. $\left(1\dfrac{1}{3}\right)\left(2\dfrac{2}{5}\right) =$ _____

16. $\dfrac{1}{10} \div \dfrac{3}{4} =$ _____

17. $\dfrac{2}{3} \div \dfrac{4}{5} =$ _____

18. $\dfrac{4}{3} \div 2 =$ _____

19. $5 \div \dfrac{4}{9} =$ _____

20. $\left(1\dfrac{2}{3}\right) \div \left(\dfrac{1}{8}\right) =$ _____

21. $\left[\dfrac{1}{2} + \dfrac{2}{7}\right] \div \left[\dfrac{4}{5} - \dfrac{1}{2}\right] =$ _____

Practice Answers

1. $\dfrac{2}{3} + \dfrac{1}{3} =$ Since the denominator is the same, add   =1

    the numerators. $\dfrac{2}{3} + \dfrac{1}{3} = \dfrac{3}{3} \, or \, 1$

2. $\dfrac{3}{4} + \dfrac{1}{6} =$ $\dfrac{9}{12} + \dfrac{2}{12} = \dfrac{11}{12}$ $= \dfrac{11}{12}$

3. $\dfrac{11}{12} - \dfrac{6}{12} =$ Since the denominator is the same, subtract

    the numerators. $\dfrac{11}{12} - \dfrac{6}{12} = \dfrac{5}{12}$ $= \dfrac{5}{12}$

4. $\dfrac{4}{5} - \dfrac{2}{3} =$ $\dfrac{12}{15} - \dfrac{10}{15} = \dfrac{2}{15}$ $= \dfrac{2}{15}$

5. $\left(\dfrac{1}{2} + \dfrac{1}{3} \right) - \dfrac{5}{12}$ $\left(\dfrac{6}{12} + \dfrac{4}{12} \right) - \dfrac{5}{12} = \dfrac{5}{12}$ $= \dfrac{5}{12}$

6. $\left(\dfrac{3}{4} - \dfrac{1}{5} \right) + \dfrac{3}{10}$ $\left(\dfrac{15}{20} - \dfrac{4}{20} \right) + \dfrac{6}{20} = \dfrac{17}{20}$ $= \dfrac{17}{20}$

7. $\dfrac{2}{9} + \dfrac{1}{3} =$ $\dfrac{2}{9} + \dfrac{3}{9} = \dfrac{5}{9}$ $= \dfrac{5}{9}$

8. $\dfrac{5}{6} - \dfrac{2}{3} =$ $\dfrac{5}{6} - \dfrac{4}{6} = \dfrac{1}{6}$ $= \dfrac{1}{6}$

9. $10 - \left(2\dfrac{1}{5} \right) =$ $\dfrac{50}{5} - \dfrac{11}{5} = \dfrac{39}{5} \, or \, 7\dfrac{4}{5}$ $= 7\dfrac{4}{5}$

10. $\left(3\dfrac{1}{2} \right) + 4 =$ $\dfrac{7}{2} + \dfrac{8}{2} = \dfrac{15}{2} \, or \, 7\dfrac{1}{2}$ $= 7\dfrac{1}{2}$

11. $\left(\dfrac{1}{11}\right)\left(\dfrac{1}{2}\right) =$ $\qquad$ $= \dfrac{1}{22}$

12. $\left(\dfrac{3}{7}\right)\left(\dfrac{2}{3}\right) =$ $\qquad$ $\dfrac{6}{21} = \dfrac{2}{7}$ $\qquad$ $= \dfrac{2}{7}$

13. $4\left(\dfrac{1}{4}\right) =$ $\qquad$ $\left(\dfrac{4}{1}\right)\left(\dfrac{1}{4}\right) = 1$ $\qquad$ $= 1$

14. $\left(\dfrac{3}{8}\right)(9) =$ $\qquad$ $\left(\dfrac{3}{8}\right)\left(\dfrac{9}{1}\right) = \dfrac{27}{8} \, or \, 3\dfrac{3}{8}$ $\qquad$ $= 3\dfrac{3}{8}$

15. $\left(1\dfrac{1}{3}\right)\left(2\dfrac{2}{5}\right) =$ $\qquad$ $\left(\dfrac{4}{3}\right)\left(\dfrac{12}{5}\right) = \dfrac{48}{15} = \dfrac{16}{5} \, or \, 3\dfrac{1}{5}$ $\qquad$ $= 3\dfrac{1}{5}$

16. $\dfrac{1}{10} \div \dfrac{3}{4} =$ $\qquad$ $\left(\dfrac{1}{10}\right)\left(\dfrac{4}{3}\right) = \dfrac{4}{30} = \dfrac{2}{15}$ $\qquad$ $= \dfrac{2}{15}$

17. $\dfrac{2}{3} \div \dfrac{4}{5} =$ $\qquad$ $\left(\dfrac{2}{3}\right)\left(\dfrac{5}{4}\right) = \dfrac{10}{12} = \dfrac{5}{6}$ $\qquad$ $= \dfrac{5}{6}$

18. $\dfrac{4}{3} \div 2 =$ $\qquad$ $\left(\dfrac{4}{3}\right)\left(\dfrac{1}{2}\right) = \dfrac{4}{6} = \dfrac{2}{3}$ $\qquad$ $= \dfrac{2}{3}$

19. $5 \div \dfrac{4}{9} =$ $\qquad$ $\left(\dfrac{5}{1}\right)\left(\dfrac{9}{4}\right) = \dfrac{45}{4} \, or \, 11\dfrac{1}{4}$ $\qquad$ $= 11\dfrac{1}{4}$

20. $\left(1\dfrac{2}{3}\right) \div \left(\dfrac{1}{8}\right) =$ $\qquad$ $\left(\dfrac{5}{3}\right)\left(\dfrac{8}{1}\right) = \dfrac{40}{3} \, or \, 13\dfrac{1}{3}$ $\qquad$ $= 13\dfrac{1}{3}$

21. $\left[\dfrac{1}{2} + \dfrac{2}{7}\right] \div \left[\dfrac{4}{5} - \dfrac{1}{2}\right] =$ $\qquad$ $\left[\dfrac{7}{14} + \dfrac{4}{14}\right] \div \left[\dfrac{8}{10} - \dfrac{5}{10}\right]$

$$= \left(\frac{11}{14}\right) \div \left(\frac{3}{10}\right)$$

$$= \left(\frac{11}{14}\right)\left(\frac{10}{3}\right)$$

$$= \left(\frac{110}{42}\right)$$

$$= \left(\frac{55}{21}\right)$$

$$= 2\frac{13}{21}$$