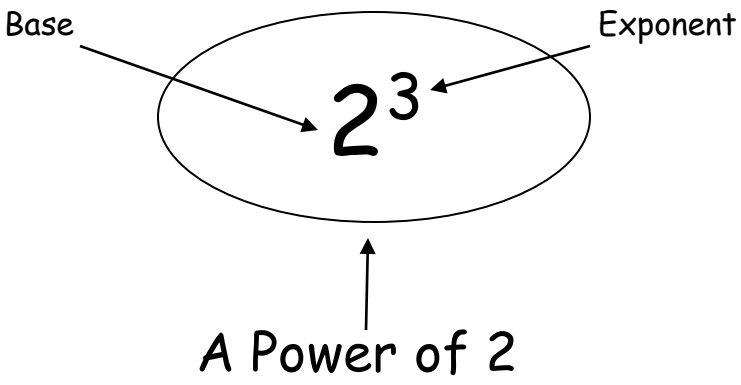
Powers and Exponents



This means: 2 X 2 X 2 And it has a value of 8

Exponent Laws:

1.Product $2^3 \times 2^4 = 2^7$ **[Keep the base, and add exponents]**

2.Quotient $\frac{2^5}{2^3} = 2^2$ **[keep the base, and subtract exponents]**

Laws with an exponent outside of brackets. Multipy the exponent outside of the brackets by every exponent on every base inside of brackets.

3. Power of a power $(2^3)^4 = 2^{12}$

4. Power of a product $(2m^4)^3 = 2^3m^{12}$ or $8m^{12}$

5. Power of a quotient $\left(\frac{2^2 m^5}{k^3}\right)^6 = \left(\frac{2^{12} m^{30}}{k^{18}}\right)$ or $\frac{4096m^{30}}{k^{18}}$

Zero exponent. Any base raised to an exponent of zero, is one.

6. Zero Exponent $2^0 = 1$

$2m^0 = 2$

$-w^0 = -1$

7. Negative exponents

- 2^{-1} = $\frac{1}{2^1}$ = $\frac{1}{2}$
- $\frac{1}{2^{-1}}$ = $\frac{2^1}{1}$ = 2
- $\frac{2^2}{2^{-4}}$ = $\frac{(2^2)(2^4)}{1} = 2^6$ =64
- $\frac{2^{-3}}{2^2}$ = $\frac{1}{(2^3)(2^2)} = \frac{1}{2^5}$ = $\frac{1}{32}$
- $\frac{2^{-4}}{2^{-7}}$ = $\frac{2^7}{2^4} = 2^3$ = 8

In each of the above examples, when a power with a negative exponent is moved from the numerator to the denominator, or the denominator to the numerator, the value of the exponent becomes POSITIVE.

- $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2$ = $\frac{9}{4}$
- $\left(\frac{1}{4}\right)^{-5} = \left(\frac{4}{1}\right)^5$ = 1024

Given a fraction with a negative exponent, reciprocate the base and the exponents becomes positive.

Practice Questions

For each of the following questions, simplify using the exponent laws. First identify which law, or laws will be used, and then simplify or evaluate.

	Question	Law(s)	Simplification
1.	$\frac{(3^3)(3^5)}{(3^2)(3^4)}$	_____	
2.	$(4k^3)^2$	_____	
3.	$(2m^2)^3\ (2m^4)^2$	_____	
4.	$\frac{(6x^2y^3)^2}{3xy^4}$	_____	
5.	$(4w^{10})\ (2w^{-4})$	_____	
6.	$\frac{12v}{v^{-2}}$	_____	
7.	$\frac{50x^3y^7}{(5xy)^2}$	_____	
8.	$\frac{1}{(m^{-3})(m^{-1})}$	_____	
9.	$\frac{24r^9k^{11}}{-4rk^2}$	_____	
10.	$\frac{(9xy^4)^2}{3xy^5}$	_____	

Working With Negative Exponents

Simplify all questions with positive exponents. [**Note; if the coefficient is negative, it stays negative; it does not change from moving denominator to numerator, or vice versa]**

1. a) $\frac{2x^3}{x^{-2}}$ b) $\frac{7y^{-5}}{y}$

2. a) $\frac{10w^{-2}}{-2w^{-5}}$ b) $\frac{-12m^{-6}}{-4m^{-1}}$

3. a) $(4x^{-2}y^3)^{-1}$ b) $(3m^9)^{-2}\,(9m^{-6})$

4. a) $\left(\frac{2}{3}\right)^{-2}\left(\frac{4}{9}\right)$ b) $\left(\frac{1}{2}\right)^{-3}\left(2^{-1}\right)$

5. a) $\frac{(7x^2)(4x^{-5})}{14x^{-4}}$ b) $\frac{(-18m^4n)}{(2m^{-1}n^3)^3}$

6. a) $\left(\frac{3x}{y}\right)^{-2}\left(\frac{y^2}{27x}\right)^{-1}$ b) $\left(\frac{4m}{3}\right)^{-4}\left(\frac{3}{8m^2}\right)^{-2}$

7. a) $\frac{(3w^2)^{-1}}{(9w^{-1})^{-2}}$ b) $\frac{-5vw^{-4}}{v^{-3}w}$

Practice Questions Answers

For each of the following questions, simplify using the exponent laws. First identify which law, or laws will be used, and then simplify or evaluate.

	Question	Law(s)	Simplification
1.	$\frac{(3^3)(3^5)}{(3^2)(3^4)}$	1 & 2	$3^2 = 9$
2.	$(4k^3)^2$	4	$16k^6$
3.	$(2m^2)^3\ (2m^4)^2$	4 & 1	$32m^{14}$
4.	$\frac{(6x^2y^3)^2}{3xy^4}$	4 & 2	$12x^3y^2$
5.	$(4w^{10})\ (2w^{-4})$	1	$8w^6$
6.	$\frac{12v}{v^{-2}}$	7 & 1	$12v^3$
7.	$\frac{50x^3y^7}{(5xy)^2}$	4 & 2	$2xy^5$
8.	$\frac{1}{(m^{-3})(m^{-1})}$	7 & 1	m^4
9.	$\frac{24r^9k^{11}}{-4rk^2}$	2	$-6r^8k^9$
10.	$\frac{(9xy^4)^2}{3xy^5}$	4 & 2	$27xy^3$

Working With Negative Exponents

Simplify all questions with positive exponents. [**Note; if the <u>coefficient</u> is negative, it stays negative; it does not change from moving denominator to numerator, or vice versa**]

11. a) $\frac{2x^3}{x^{-2}} = 2x^5$ b) $\frac{7y^{-5}}{y} = \frac{7}{y^6}$

12. a) $\frac{10w^{-2}}{-2w^{-5}} = -5w^3$ b) $\frac{-12m^{-6}}{-4m^{-1}} = \frac{3}{m^5}$

13. a) $(4x^{-2}y^3)^{-1} = \frac{x^2}{4y^3}$ b) $(3m^9)^{-2}(9m^{-6}) = \frac{1}{m^{24}}$

14. a) $\left(\frac{2}{3}\right)^{-2}\left(\frac{4}{9}\right) = 1$ b) $\left(\frac{1}{2}\right)^{-3}\left(2^{-1}\right) = 4$

15. a) $\frac{(7x^2)(4x^{-5})}{14x^{-4}} = 2x$ b) $\frac{(-18m^4n)}{(2m^{-1}n^3)^3} = \frac{-9m^7}{4n^8}$

16. a) $\left(\frac{3x}{y}\right)^{-2}\left(\frac{y^2}{27x}\right)^{-1}$

$= \left(\frac{y}{3x}\right)^{2}\left(\frac{27x}{y^2}\right)^{1}$

$= \left(\frac{y^2}{9x^2}\right)\left(\frac{27x}{y^2}\right)$

$= \frac{3}{x}$

b) $\left(\frac{4m}{3}\right)^{-4}\left(\frac{3}{8m^2}\right)^{-2}$

$= \left(\frac{3}{4m}\right)^{4}\left(\frac{8m^2}{3}\right)^{2}$

$= \left(\frac{3^4}{4^4 m^4}\right)\left(\frac{8^2 m^4}{3^2}\right)$

$= \left(\frac{81}{256m^4}\right)\left(\frac{64m^4}{9}\right)$

$= 2.25$

17. a) $\frac{(3w^2)^{-1}}{(9w^{-1})^{-2}}$

$= \left(\frac{3^{-1}w^{-2}}{9^{-2}w^2}\right)$

$= \frac{9^2}{3^1(w^2)(w^2)}$

$= \frac{27}{w^4}$

b) $\frac{-5vw^{-4}}{v^{-3}w}$

$= \frac{-5(v)(v^3)}{(w^4)(w)}$

$= \frac{-5v^4}{w^5}$