Analyzing Rational FunctionsSolutions

Use the information below to answer the first question.

f(x) = $\dfrac{x+2}{x^2-3x-10}$ is a rational function.

1. Which statement below is true?

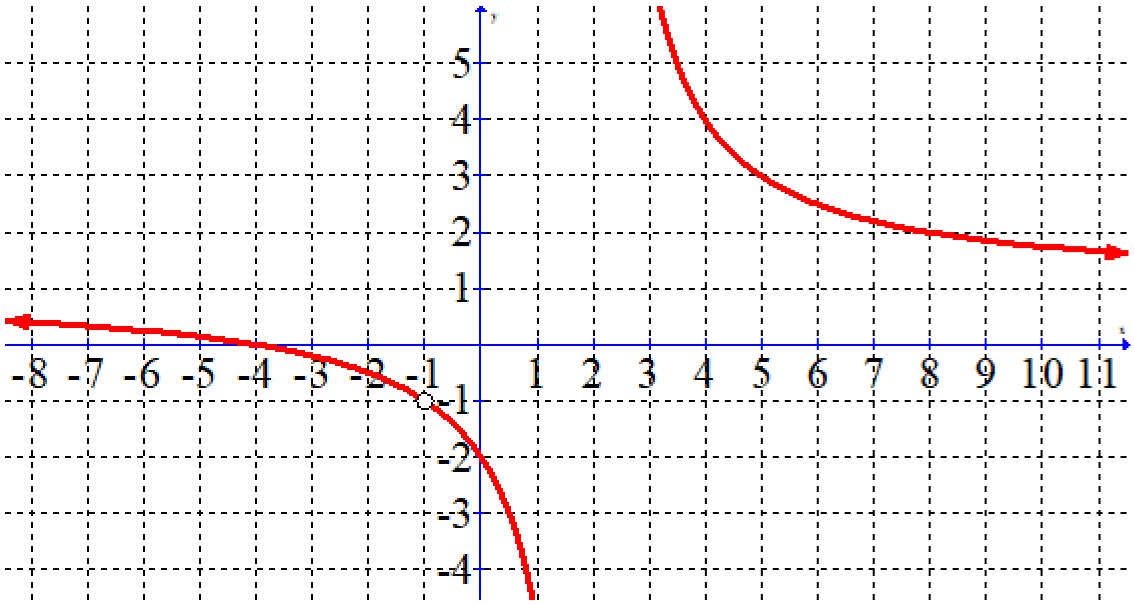
Re-write the equation in factored form.

f(x) = $\dfrac{x+2}{(x-5)(x+2)}$

a) The domain is x ≠ -5. False; x ≠ 5 and -2
b) The range is y > 0. False; y ≠ 0
c) The point of discontinuity is $\left(-2, \frac{1}{7}\right)$. False. The y-coordinate is $-\frac{1}{7}$
d) The horizontal asymptote is y = 0. True

Use the information below to answer the next question.

The equation of the vertical asymptote of the graph below is x = 2 and a point of discontinuity is at (-1,-1)



2. Which equation best describes the graph?

   a) $y = \frac{(x+4)(x+1)}{(x-2)(x+1)}$ Answer

   b) $y = \frac{(x+4)}{(x-2)}$

   c) $y = \frac{(x+4)}{(x+2)(x+1)}$

   d) $y = \frac{(x+4)(x+3)}{(x-2)(x+3)}$

The graph has a point of discontinuity at (-1,-1). Of the choices listed above, answer b) cannot describe the graph because there is only 1 different binomial in the numerator and the denominator.

Answer c) above, does not have a common binomial in the numerator and the denominator to divide out.

Both answers a) and d) have common binomials. For d), once the common (x + 3) is divided out of the numerator and the denominator, the point of discontinuity will have an x-coordinate of x = -3. The graph shows that the point of discontinuity must have an x-coordinate of x = -1.

For answer a), once the common binomial of (x + 1) is divided out, this point of discontinuity will have the required value of x, as x = -1.

3. The rational function, f(x) = $\frac{(4x+1)(x-k)}{(x-5)(x-k)}$ has a point of discontinuity at

a) $\left(-k, \frac{4k+1}{k-5}\right)$

b) $\left(k, \frac{4k+1}{k-5}\right)$ Answer

c) $\left(-x, \frac{4k+1}{k-5}\right)$

d) $\left(x, \frac{k-5}{4k+1}\right)$

The common binomial to divide out of the numerator and the denominator is (x – k). The question here is, what value of x will make this binomial equal to zero. If x = k, then k – k = 0. Thus, the x-coordinate of the point of discontinuity is **k**.

Substitute **k** into the simplified equation, f(x) $\frac{(4x+1)}{(x-5)}$ = to find the y-coordinate.

f(x) = $\frac{(4(k)+1)}{((k)-5)}$

f(x) = $\frac{(4k+1)}{(k-5)}$

Use the following information to answer the next question.

Consider two graphs:

f(x) = $\frac{x^2 - 2x - 3}{x+3}$ and g(x) = $\frac{x^2 + 2x - 3}{x+3}$

4. The graph having a vertical asymptote is _f(x)__ and the graph having a point of discontinuity is _g(x)_. Each has a non-permissible value of x ≠ -3

The factored form of f(x) is $\frac{(x-3)(x+1)}{(x+3)}$. Since there is no common binomial to divide out of the numerator and the denominator, this equation will have a vertical asymptote, but no point of discontinuity.

The factored form of g(x) is $\frac{(x+3)(x-1)}{(x+3)}$. Since there is a common binomial to divide out of the numerator and the denominator, there is a point of discontinuity.

Both equations have the same binomial denominator of (x + 3). The value of x which would make this denominator equal to zero is -3. Therefore the non-permissible value for each equation is x ≠ -3.

Use the following information to answer the next question.

Given the function f(x) = $\frac{(4x-3)(5x+1)}{(-x+2)(4x-3)}$, the possible equations for the vertical and horizontal asymptotes are listed in the chart below.

	Vertical Asymptote	Horizontal Asymptote
A	x = $\frac{3}{4}$	y = -5
B	x = $\frac{3}{4}$	y = 4
C	x = 2	y = -5
D	x = 2	y = 4

5. The row that accurately states each asymptote is row _c__

The simplified form of f(x) = $\frac{(4x-3)(5x+1)}{(-x+2)(4x-3)}$

is f(x) = $\frac{5x+1}{-x+2}$, when the common (4x – 3) is divided out.

If the degree of each polynomial in the numerator and the denominator is the same (as in this case with each being a first degree), the horizontal asymptote is determined by dividing the coefficients on the lettered terms.

Thus, 5/-1 is -5. **The equation of the horizontal asymptote is y = -5.**

The vertical asymptote is determined by analyzing the denominator. If x = 2, the denominator will be equal to zero, and therefore, undefined. **The equation of the vertical asymptote is x = 2.**

6. The rational equation f(x) = $\frac{ax}{x^2-k}$ has a domain of x ≠ ± 6. If f(3) = -1, then the value of k is _36__ and the value of a is __9___.

To have a domain of x ≠ ± 6, the denominator would have to be (x – 6) (x + 6). The product of these 2 binomials is x^2 – 36.

Therefore, k = 36

Substitute the point (3,-1) into the equation to find the value of **a**.

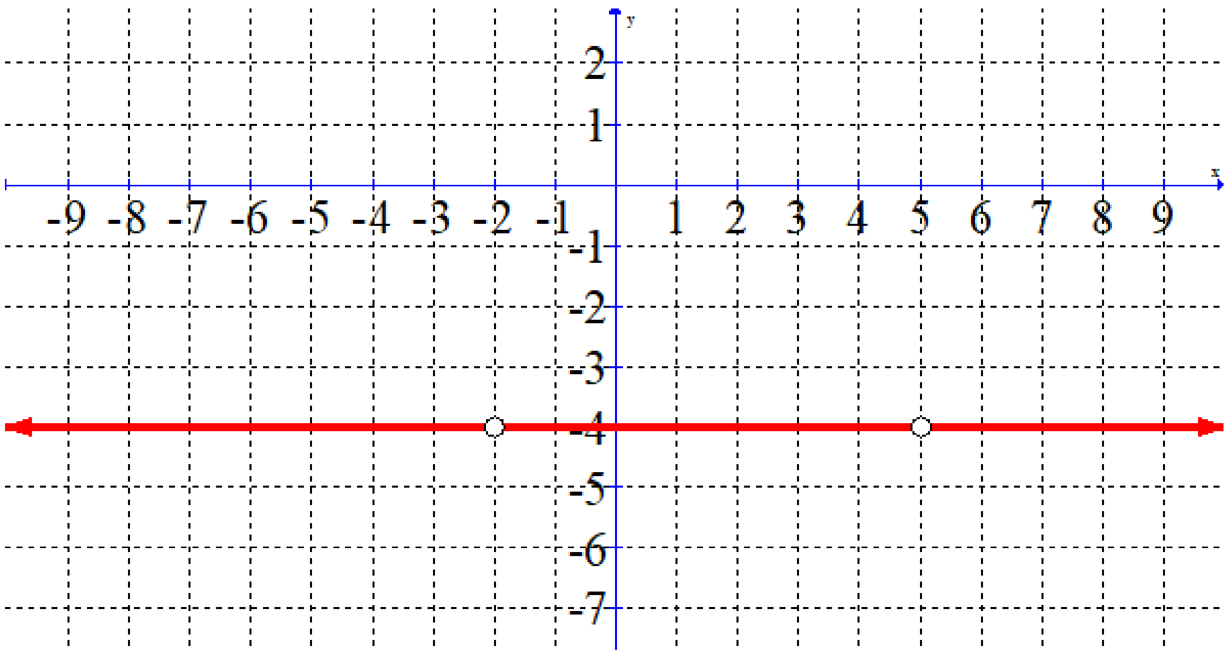
$$-1=\frac{a(3)}{(3)^2-36}$$

$$-1=\frac{3a}{9-36}$$

27 = 3a

a = 9

7. What is the equation of the rational function below?



There are 2 points of discontinuity, at (-2,-4) and (5,-4). This means that there would have been 2 binomials to be divided out of the numerator and the denominator. The x-coordinates of -2 and 5 from these points of discontinuity, would have made the equation undefined. The binomials to represent these numbers are (x + 2) and (x – 5).

Since the equation of the line on the graph above is y = -4, the equation of the original rational function would be $y = \frac{-4(x+2)(x-5)}{(x+2)(x-5)}$.