Proving Identities Solutions

Refer to the list of strategies below when answering the following 4 questions.

1. Use formula sheet substitutions.
2. Convert to sine and cosine ratios.
3. Use the conjugate.
4. Use factoring.
5. Combine 2 terms into 1 term.

1. A possible first step to simplifying $\frac{\sin^2 x}{1+\cos x}$ is $\frac{\sin^2 x}{1+\cos x}\left(\frac{1-\cos x}{1-\cos x}\right)$.

   Identify the strategy used, and complete the simplification.

The strategy is #3, using the conjugate.

= $\frac{\sin^2 x - \sin^2 x \cos x}{1-\cos^2 x}$

Substitute $\sin^2 x$ for $(1 - \cos^2 x)$ and factor out a $\sin^2 x$ from the terms on the numerator.

= $\frac{\sin^2 x(1-\cos x)}{\sin^2 x}$

= 1 - cosx

2. In order to first simplify the numerator of $\frac{4\cos^2 x - 1}{2\cos^2 x + 1}$, identify the strategy that would likely be used. Complete the simplification.

The strategy is #4, factoring.

$$\frac{(2\cos^2 x+1)(2\cos^2 x-1)}{2\cos^2 x+1}$$

$= 2\cos^2 x - 1$

3. Consider the expression, $\frac{\cot x}{\cos^3 x+\sin^2 x\cos x}$.
   a) What strategy could be used on the numerator?

Strategy #2. Re-write cotx as $\frac{\cos x}{\sin x}$

   b) Which 2 strategies, in order, could be used on the denominator?

Strategy #4. Factor out a common cosx.

$\cos x(\cos^2 x + \sin^2 x)$

Strategy #1. Substitute from the formula sheet, 1 for $(\cos^2 x + \sin^2 x)$

   c) Simplify the expression.

$$\frac{\frac{\cos x}{\sin x}}{\cos x}$$

$$= \left(\frac{\cos x}{\sin x}\right)\left(\frac{1}{\cos x}\right)=\frac{1}{\sin x}=\csc x$$

4. Given $\frac{\sec x+\sin x}{1+\sin x\cos x}$, which strategy can be used on the numerator? Complete the simplification.

Re-write secx as $\frac{1}{\cos x}$ and then add $\frac{1}{\cos x}$ to sinx to put them into 1 term.

$$\frac{1}{\cos x}+\frac{\sin x\cos x}{\cos x}$$

= $\dfrac{1+\sin x\cos x}{\cos x}$

To complete the simplification, write this over the denominator in the original equation.

= $\dfrac{\dfrac{1+\sin x\cos x}{\cos x}}{1+\sin x\cos x}$

= $\dfrac{1+\sin x\cos x}{\cos x} X \dfrac{1}{1+\sin x\cos x}$

= $\dfrac{1}{\cos x} = \sec x$

5. The expression, $\dfrac{\tan x - \sec x}{1-\sin x}$, for all permissible values of x, is equivalent to

a) csc x b) -csc x c) sec x d) -sec x

$\dfrac{\dfrac{\sin x}{\cos x} - \dfrac{1}{\cos x}}{1-\sin x}$

= $\dfrac{\dfrac{\sin x - 1}{\cos x}}{1-\sin x}$

= $\dfrac{\sin x - 1}{\cos x} X \dfrac{1}{-1(\sin x - 1)}$

= $-\dfrac{1}{\cos x} = -\sec x$

6. a) Given the identity, $\frac{2\cos^2\theta - 2}{\sin\theta} = -2\sin\theta$, use an algebraic process to show LS = RS.

Begin with the left side, which is the more complex side.

Factor out a 2 from the two terms on the numerator.

$$\frac{2(\cos^2\theta - 1)}{\sin\theta}$$

Substitute (1 – $\sin^2\theta$) for ($\cos^2\theta$)

$$= \frac{2((1-\sin^2\theta)-1)}{\sin\theta}$$

$$= \frac{-2\sin^2\theta}{\sin\theta}$$

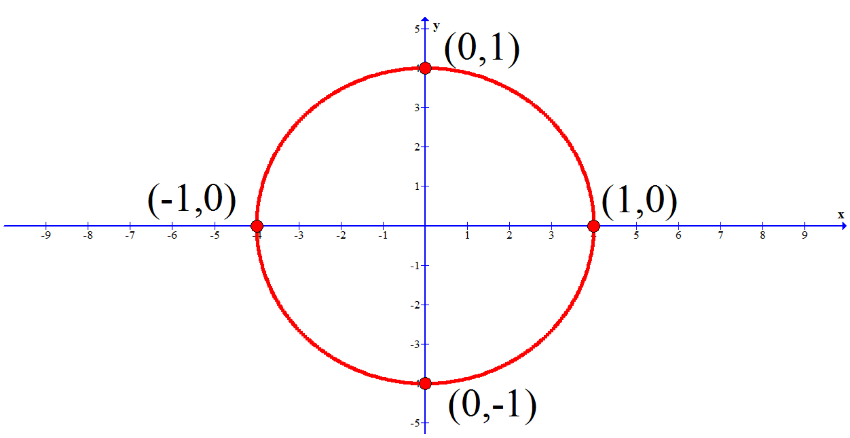
= -2sinθ

Left side = Right side

b) Which non-permissible values for θ should be stated for this identity?

sinθ≠0

Using the graph below, sinπ is 0, and then it occurs every π. Thus, the non-permissible values are θ ≠ πn, n ∈ I.



7. Use double-angle identities to prove $\frac{2\tan x}{1-\tan^2 x} = \frac{\sin(2x)}{\cos^2 x - \sin^2 x}$

$$\tan 2x = \frac{\sin 2x}{\cos 2x}$$

tan 2x = tan 2x

8. Is, $\frac{\sin(2x)}{1+\cos(2x)} = \cot x$, an identity? Explain.

$$\frac{\sin(2x)}{1+\cos(2x)} = \cot x$$

Substitute the double-angle identities for sin(2x) and cos(2x) from the formula sheet.

<u>Left Side</u>

$$= \frac{2\sin x\cos x}{1+\cos^2 x - \sin^2 x}$$

$1 = \cos^2 x + \sin^2 x$

$$= \frac{2\sin x\cos x}{\cos^2 x + \sin^2 x + \cos^2 x - \sin^2 x}$$

Simplify the denominator by combining like terms.

$$= \frac{2\sin x\cos x}{2\cos^2 x}$$

$$= \frac{\sin x}{\cos x}$$

= tan x

Since tan x ≠ cot x, this is not an identity.

9. Prove the identity $\frac{\sin 2x - \cos x}{4\sin^2 x - 1} = \frac{\sin^2 x\cos x + \cos^3 x}{2\sin x + 1}$ for all permissible values of x.

= $\frac{2\sin x\cos x - \cos x}{(2\sin x + 1)(2\sin x - 1)} = \frac{\cos x(\sin^2 x + \cos^2 x)}{2\sin x + 1}$

= $\frac{\cos x(2\sin x - 1)}{(2\sin x + 1)(2\sin x - 1)} = \frac{\cos x}{2\sin x + 1}$

= $\frac{\cos x}{2\sin x + 1} = \frac{\cos x}{2\sin x + 1}$