

Math 30-2 Counting Methods Lesson 5 Practice Questions [Solutions at the end]

Use the following information to answer the first question.

A student is classifying the following contexts as either permutations or combinations.

Context A	Choosing a committee of 3 people to be president, vice-president and treasurer.
Context B	Selecting 3 fruits to put in a smoothie.
Context C	Determining when to add a new area code.
Context D	Finding the number of triangles that can be drawn in a pentagon.

1. For each context, use **1** to indicate that the context would be classified as a permutation and use a **2** to indicate that the context would be classified as a combination.

Context A \_\_\_\_ Context B \_\_\_\_ Context C \_\_\_\_ Context D \_\_\_\_

Use the following information to answer the next question.

There are 21 females and 26 males in a graduating grade 12 class. The school's graduation advisor wants to select a grad committee of 10 students that has the same number of males as females. The number of possible committees can be determined by:

$${}_{21}C_m \times {}_{26}C_n$$

2. The value of  $m + n$  is \_\_\_\_.

Use the following information to answer the next question.

A readings list for a Humanities course consists of 10 books, of which 4 are biographies and the rest are novels. Each student is required to read a selection of 4 books from the list, including at least 2 biographies.

3. Which of the following calculations could be used to determine the number of 4 book selections?

- A)  $({}^4C_2)({}^6C_2) \times ({}^4C_3)({}^6C_1) \times ({}^4C_4)$
- B)  $({}^4C_2)({}^6C_2) + ({}^4C_3)({}^6C_1) + ({}^4C_4)$
- C)  $({}^{10}C_4) \times ({}^{10}C_6)$
- D)  $({}^{10}C_4) + ({}^{10}C_6)$

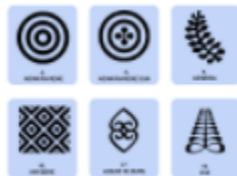
Use the following information to answer the next question.

The CEO of a construction company wants to form a committee of 5 managers (from a total of 5 females and 4 males) to investigate options for future growth. Jacinta needs to be on the committee because she has a wealth of experience in this area. The CEO has stated that there must be 3 females and 2 males.

4. The number of possible committees is \_\_\_\_\_.

Use the following information to answer the next question.

The following symbols are selected and arranged to create various "code" words. No symbol can be repeated in a code word.



5. Which of the following calculations can be used to determine the number of possible code words that can be created when **at least** 4 symbols are selected and arranged?

- A)  ${}^6P_4 \cdot {}^6P_5 \cdot {}^6P_6$
- B)  ${}^6P_4 + {}^6P_5 + {}^6P_6$
- C)  ${}^6C_4 \cdot {}^6C_5 \cdot {}^6C_6$
- D)  ${}^6C_4 + {}^6C_5 + {}^6C_6$

Use the following information to answer the next question.

The 8 croquet balls shown below can all fit in a carrying bag.



6. If all of these balls were in the carrying bag and you reached in to randomly select 4 balls, how many possible ways are there to draw **at most** 1 yellow ball?

Use the following information to answer the next question.

In the expression below, the value of  $r$  is 2.

$$\binom{n}{r} = 45$$

7. The value of  $n$  is \_\_\_\_\_.

Use the following information to answer the next question.

Allan just joined an online video store that offers 4 free movie downloads when registering. There is a collection of 80 movies to choose from, including 20 Sci Fi, 28 Comedies and 32 Dramas. Allan has no interest in Sci Fi and is thinking that he would like to get 3 Dramas and 1 Comedy.

8. The number of ways Allan can select his free movies is

- A) 3840                      B) 4960                      C) 138 880                      D) 1 581 580

9. A volleyball league has two divisions of 6 teams each. During the regular season, teams play only within their division, and play each team twice. An expression for the total number of regular season games is

- A)  $2({}_6C_2) + 2({}_6C_2)$   
B)  $2({}_6C_2) \cdot 2({}_6C_2)$   
C)  ${}_6C_4 + {}_6C_4$   
D)  ${}_6C_4 \cdot {}_6C_4$

Math 30-2 Counting Methods Lesson 5 Practice Questions **Solutions**

Use the following information to answer the first question.

A student is classifying the following contexts as either permutations or combinations.

Context A	Choosing a committee of 3 people to be president, vice-president and treasurer.
Context B	Selecting 3 fruits to put in a smoothie.
Context C	Determining when to add a new area code.
Context D	Finding the number of triangles that can be drawn in a pentagon.

1. For each context, use **1** to indicate that the context would be classified as a permutation and use a **2** to indicate that the context would be classified as a combination.

Context A   1      Context B   2      Context C   1      Context D   2  

**Solution**

Usually committee selection deals with combinations. As soon as specific positions are defined within the committee, as we see here, the application of permutations is required. If Joanna is president, Bobby VP and Julia treasurer, then a different permutation is represented by Bobby president, Julia VP and Joanna treasurer. This context is a permutation, or #1.

The order of 3 fruits to be added to a smoothie does not matter. Adding blueberries, strawberries and a banana will taste the same regardless of when the fruits were added. This context is a combination, or #2.

Determining when to add a new area code for telephone numbers means that order is important. Each specific order of a set of numbers will produce a unique telephone number. This context is a permutation, or #1.

Within a pentagon ( a 5 sided polygon), triangles can be formed by joining any 3 vertices. One group of 3 vertices produces only one triangle; the order of the

vertices chosen has no bearing on the number of triangles formed. This context is a combination, or #2.

Use the following information to answer the next question.

There are 21 females and 26 males in a graduating grade 12 class. The school's graduation advisor wants to select a grad committee of 10 students that has the same number of males as females. The number of possible committees can be determined by:

$${}_{21}C_m \times {}_vC_n$$

2. The value of  $m + v$  is 31.

**Solution**

The committee should consist of 5 males and 5 females. There are two stages:

Stage 1 - Groups of Females

$${}_{21}C_5$$

X

Stage 2 - Groups of Males

$${}_{26}C_5$$

The value of  $m$  is 5 and the value of  $v$  is 26.

The value of  $m + v$  is 31.

Use the following information to answer the next question.

A readings list for a Humanities course consists of 10 books, of which 4 are biographies and the rest are novels. Each student is required to read a selection of 4 books from the list, including at least 2 biographies.

3. Which of the following calculations could be used to determine the number of 4 book selections?

A)  $(4C_2)(6C_2) \times (4C_3)(6C_1) \times (4C_4)$

B)  $(4C_2)(6C_2) + (4C_3)(6C_1) + (4C_4)$

C)  $(10C_4) \times (10C_6)$

D)  $(10C_4) + (10C_6)$

### Solutions

There are 3 cases that need to be determined individually and then summed.

The question says that *at least* two biographies must be chosen. In this context, *at least two* means 2 or 3 or 4 autobiographies.

Case 1 - 2 Bio

Case 2 - 3 Bio

Case 3 - 4 Bio

$(4C_2)(6C_2)$

+

$(4C_3)(6C_1)$

+

$(4C_4)$

The correct answer is B.

Use the following information to answer the next question.

The CEO of a construction company wants to form a committee of 5 managers (from a total of 5 females and 4 males) to investigate options for future growth. Jacinta needs to be on the committee because she has a wealth of experience in this area. The CEO has stated that there must be 3 females and 2 males.

4. The number of possible committees is 36.

### Solution

There are three stages, Jacinta, groups of females and groups of males.

There is only 1 way to choose Jacinta. Her mandatory attendance means that 2 more females are required from a group of 4 remaining females. The other two males spots are to be chosen from the group of 4 males.

$$\underline{1} \quad \times \quad \underline{{}_4C_2} \quad \times \quad \underline{{}_4C_2}$$

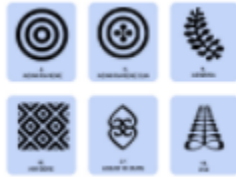
Jacinta                      Females                      Males

$$= (1) (6) (6)$$

The number of possible committees is 36.

Use the following information to answer the next question.

The following symbols are selected and arranged to create various "code" words. No symbol can be repeated in a code word.



5. Which of the following calculations can be used to determine the number of possible code words that can be created when **at least** 4 symbols are selected and arranged?

- A)  ${}_6P_4 \cdot {}_6P_5 \cdot {}_6P_6$
- B)  ${}_6P_4 + {}_6P_5 + {}_6P_6$
- C)  ${}_6C_4 \cdot {}_6C_5 \cdot {}_6C_6$
- D)  ${}_6C_4 + {}_6C_5 + {}_6C_6$

### Solution

In this context, **at least**, means 4 or 5 or 6 symbols. Determine the number of ways for each individual case, and then add these values to get the final answer. Since we are arranging the symbols, order is important. The answer is an application of permutations.

$$\underline{\text{Case 1 - 4 Symbols}} \quad + \quad \underline{\text{Case 2 - 5 Symbols}} \quad + \quad \underline{\text{Case 3 - 6 Symbols}}$$

$${}_6P_4 \quad + \quad {}_6P_5 \quad + \quad {}_6P_6$$



The correct answer is B.

Use the following information to answer the next question.

The 8 croquet balls shown below can all fit in a carrying bag.



6. If all of these balls were in the carrying bag and you reached in to randomly select 4 balls, how many possible ways are there to draw **at most** 1 yellow ball?

**Solution**

In this context, **at most** means 0 or 1 yellow balls. There are two cases which will be determined individually and then summed.

There are 2 yellow balls and 6 non-yellow balls.

$$\begin{array}{rcl} \underline{\text{Case 1 - 0 Yellow Balls}} & + & \underline{\text{Case 2 - 1 Yellow Ball}} \\ {}_6C_4 & + & ({}_6C_3)({}_2C_1) \\ 15 & + & (20)(2) \\ = & & 55 \end{array}$$

There are 55 possible ways to draw at most 1 yellow ball.

Use the following information to answer the next question.

In the expression below, the value of  $r$  is 2.

$$\binom{n}{r} = 45$$

7. The value of  $n$  is 10.

**Solution**

The notation above refers to combinations. The equivalent form is  ${}_nC_2 = 45$

Use the combination formula to re-write with factorial notation.

$$\frac{n!}{(n-2)!2!} = 45$$

Multiply both sides of the equal sign by 2!.

$$\frac{n!}{(n-2)!} = 90$$

Re-write the numerator in a form that will allow for the division of a common factorial.

$$\frac{n(n-1)(n-2)!}{(n-2)!} = 90$$

$$n(n-1) = 90$$

Set equal to zero, as we are solving a quadratic equation.

$$n^2 - n - 90 = 0$$

Factor and use the Zero Product Property.

$$(n-10)(n+9) = 0$$

$$n = 10 \text{ and } -9$$

Since -9 does not make sense in this context, the solution is 10.

Use the following information to answer the next question.

Allan just joined an online video store that offers 4 free movie downloads when registering. There is a collection of 80 movies to choose from, including 20 Sci Fi, 28 Comedies and 32 Dramas. Allan has no interest in Sci Fi and is thinking that he would like to get 3 Dramas and 1 Comedy.

8. The number of ways Allan can select his free movies is

A) 3840

B) 4960

C) 138 880

D) 1 581 580

**Solution**

$$({}_{28}C_1) ({}_{32}C_3) = 138\,880$$

9. A volleyball league has two divisions of 6 teams each. During the regular season, teams play only within their division, and play each team twice. An expression for the total number of regular season games is

A)  $2({}_6C_2) + 2({}_6C_2)$

B)  $2({}_6C_2) \cdot 2({}_6C_2)$

C)  ${}_6C_4 + {}_6C_4$

D)  ${}_6C_4 \cdot {}_6C_4$

**Solution**

One game means that two teams are combined. With a total of 6 teams in a division, we want the number of ways 6 teams can make combinations of two teams. This is determined with  ${}_6C_2$ . To play each other twice, would be double this amount, or  $2({}_6C_2)$ .

The expression  $2({}_6C_2)$  will determine the total number of games in one division. Since there are two divisions, this number is added to itself.

The correct answer is A.