Equations of Perpendicular Lines Practice

1. Determine the equation of the line perpendicular to $y = -\frac{5}{4}x - 9$ and passing through the point (1,5).
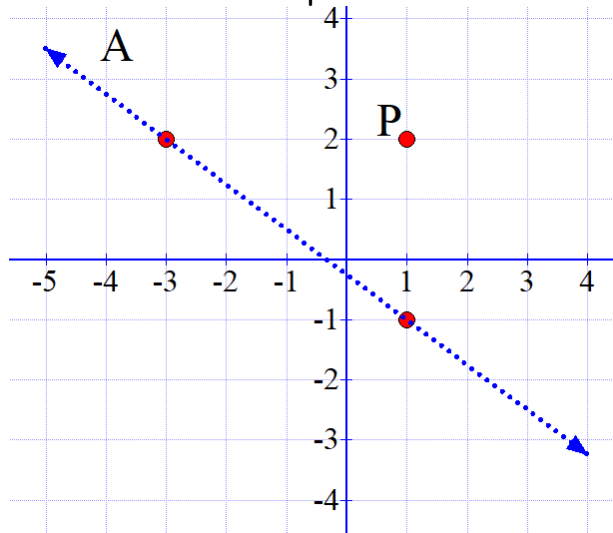
   A) $y = -\frac{4}{5}x + 9$ 　　B) $y = \frac{4}{5}x + \frac{21}{5}$ 　　C) $y = \frac{4}{5}x + \frac{29}{5}$ 　　D) $y = -\frac{4}{5}x + \frac{17}{4}$

2. A math student was asked to determine the equation of the line perpendicular to 2x + y = 10 and passing through the point (-4,-7), and put it in the form y = mx – b. The slope and y-intercept of the line perpendicular to 2x + y = 10 are

   A) slope = -2 and y-intercept = -5

   B) slope = -2 and y-intercept = -10

   C) slope = $\frac{1}{2}$ and y-intercept = -10

   D) slope = $\frac{1}{2}$ and y-intercept = -5

Use the following information to answer the next question.

3. Find the equation of the line perpendicular to line A and passing through point P.
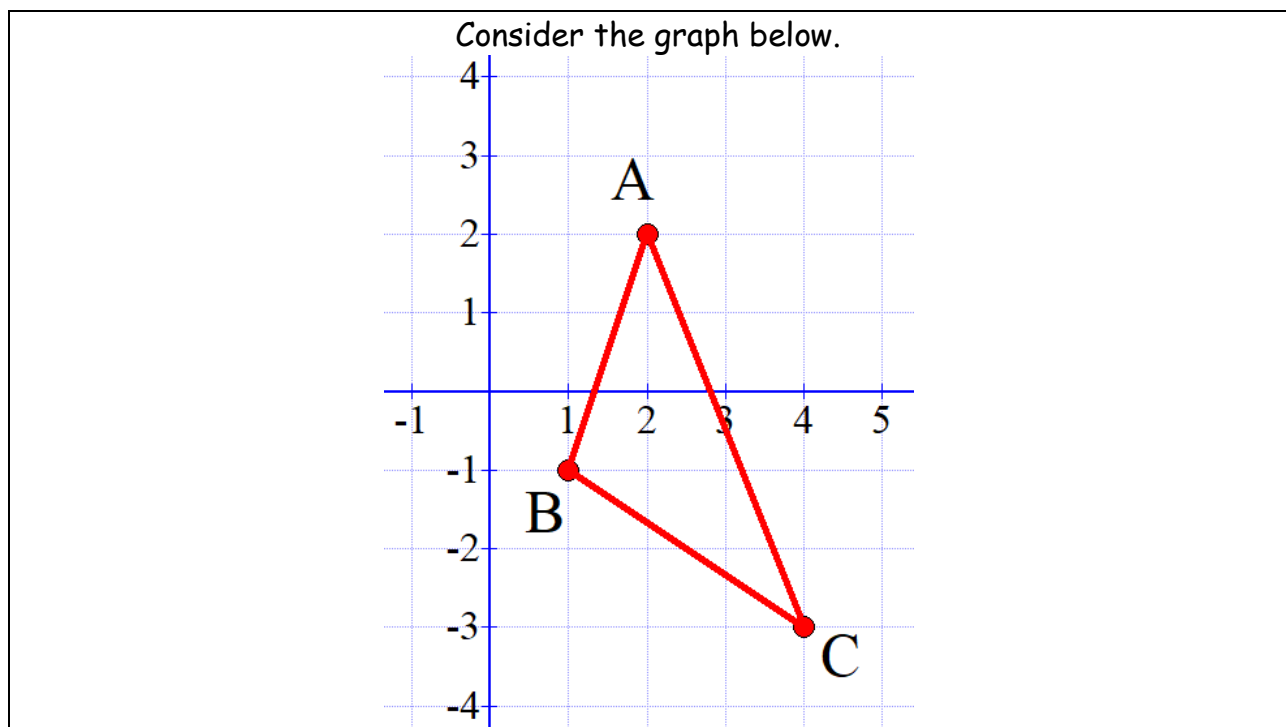
4. Which of the following pairs of lines are perpendicular?

A) $y = mx + b$ and $y = \dfrac{m}{2}x + b$

B) $y = mx + b$ and $y = -\dfrac{m}{2}x + b$

C) $y = mx + b$ and $y = \dfrac{1}{m}x + b$

D) $y = mx + b$ and $y = \dfrac{-1}{m}x + b$

Consider the graph below.

5. Is triangle ABC a right triangle? Justify.

1. Determine the equation of the line perpendicular to $y = -\dfrac{5}{4}x - 9$ and passing through the point (1,5).

A) $y = -\dfrac{4}{5}x + 9$ B) $y = \dfrac{4}{5}x + \dfrac{21}{5}$ C) $y = \dfrac{4}{5}x + \dfrac{29}{5}$ D) $y = -\dfrac{4}{5}x + \dfrac{17}{4}$

Solution

The equation $y = -\dfrac{5}{4}x - 9$ is written in the slope-intercept form. Thus, we know that the slope is $-\dfrac{5}{4}$. The slope of the line perpendicular to this line is the negative reciprocal of $-\dfrac{5}{4}$, which is $\dfrac{4}{5}$.

To determine the equation of the line perpendicular to $y = -\dfrac{5}{4}x - 9$, use the slope of $\dfrac{4}{5}$ and the given point (1,5).

$$slope = \dfrac{rise}{run}$$

$$\dfrac{4}{5} = \dfrac{y-5}{x-1}$$

Cross-multiply

4(x – 1) = 5(y – 5)

4x – 4 = 5y – 25

Add 25 to both sides

4x + 21 = 5y

Divide every term by 5.

$$y = \left(\dfrac{4}{5}\right)x + \dfrac{21}{5}$$ The correct answer is B.

2. A math student was asked to determine the equation of the line perpendicular to 2x + y = 10 and passing through the point (-4,-7), and put it in the form y = mx – b. The slope and y-intercept of the line perpendicular to 2x + y = 10 are

A) slope = -2 and y-intercept = -5
B) slope = -2 and y-intercept = -10
C) slope = $\frac{1}{2}$ and y-intercept = -10
D) slope = $\frac{1}{2}$ and y-intercept = -5

Solution

Isolate y for the given equation.

Given 2x + y = 10, subtract 2x from both sides.

y = -2x + 10

The slope of this line is -2. The slope of the line perpendicular to this line is its negative reciprocal, or $\frac{1}{2}$.

To find the y-intercept, use the slope of $\frac{1}{2}$ and the given point (-4, -7) to first find the equation.

$$slope = \frac{rise}{run}$$

$$\frac{1}{2} = \frac{y-(-7)}{x-(-4)}$$

Cross multiply

1(x + 4) = 2(y + 7)

x + 4 = 2y + 14

Subtract 14 from both sides.
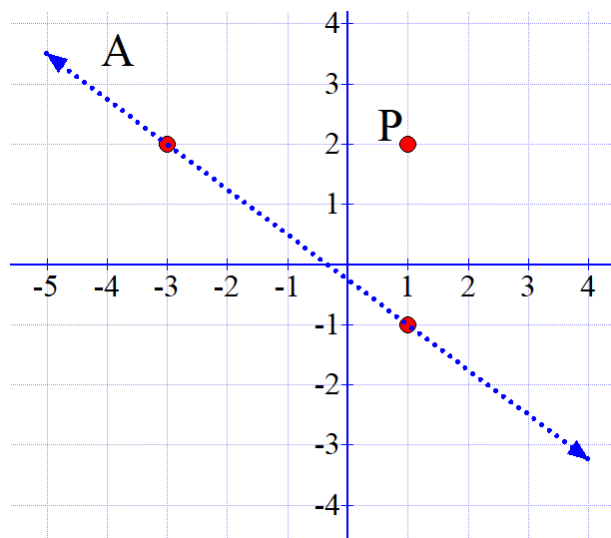
x – 10 = 2y

Divide every term by 2.

$$y = \left(\frac{1}{2}\right)x - 5$$

The y-intercept is -5.

The correct answer is D.

Use the following information to answer the next question.

3. Find the equation of the line perpendicular to line A and passing through point P.

Solution

First determine the slope of line A by using the 2 points given on the graph, (-3,2) and (1,-1).

$$slope = \frac{2 - (-1)}{-3 - 1} = \frac{3}{-4}$$

The negative reciprocal of line A is $\dfrac{4}{3}$. Therefore, the slope of the line perpendicular to line A is $\dfrac{4}{3}$.

Use point P(1,2) and the slope $\dfrac{4}{3}$ to determine the equation of the line.

$$\frac{4}{3} = \frac{y-2}{x-1}$$

Cross multiply.

4(x – 1) = 3(y – 2)

4x – 4 = 3y – 6

Add 6 to both sides.

4x + 2 = 3y

Divide every term by 3.

$$y = \left(\frac{4}{3}\right)x + \frac{2}{3}$$

Find the equation of the line perpendicular to line A and passing through point P is
$$y = \left(\frac{4}{3}\right)x + \frac{2}{3}.$$

4. Which of the following pairs of lines are perpendicular?

A) $y = mx + b$ and $y = \dfrac{m}{2}x + b$
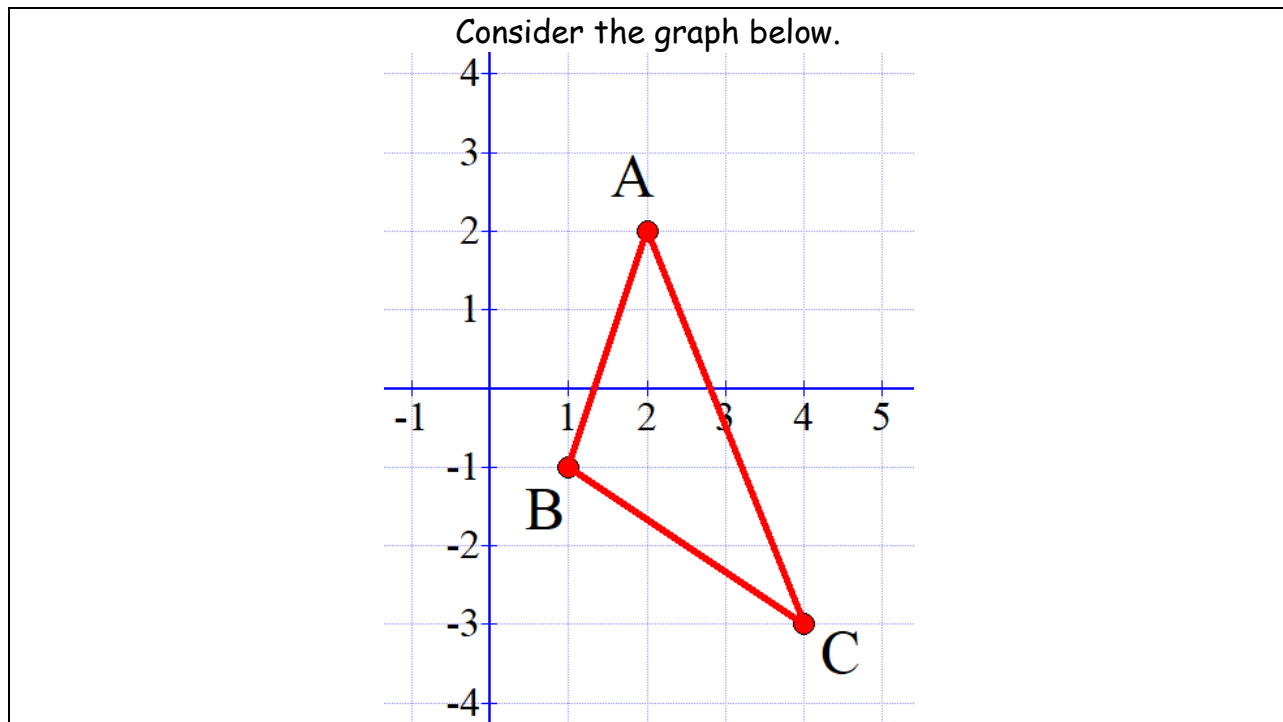
B) $y = mx + b$ and $y = -\dfrac{m}{2}x + b$

C) $y = mx + b$ and $y = \dfrac{1}{m}x + b$

D) $y = mx + b$ and $y = \dfrac{-1}{m}x + b$

The slope of $y = mx + b$ is m.

The slope of $y = \dfrac{-1}{m}x + b$ is $\dfrac{-1}{m}$

Since the product of these two slopes is -1, these lines must be perpendicular.

Consider the graph below.

5. Is triangle ABC a right triangle? Justify.

Solution

The only angle that might be 90^0 is angle B. Determine the slopes of line segments AB and BC.

From the graph, we can tell that between points A and B, the graph rises 3 units and moves horizontally 1 unit right. The slope of this line is positive and is 3/1 or 3.

From the graph, we can tell that between points C and B, the graph rises 2 units and moves horizontally 3 units left. The slope of this line is negative and is -2/3.

Multiply these slopes. $(3)\left(-\dfrac{2}{3}\right) = -2$

Since the product of these slopes is **not** -1, these lines are not perpendicular. Therefore, triangle ABC is **not** a right triangle.