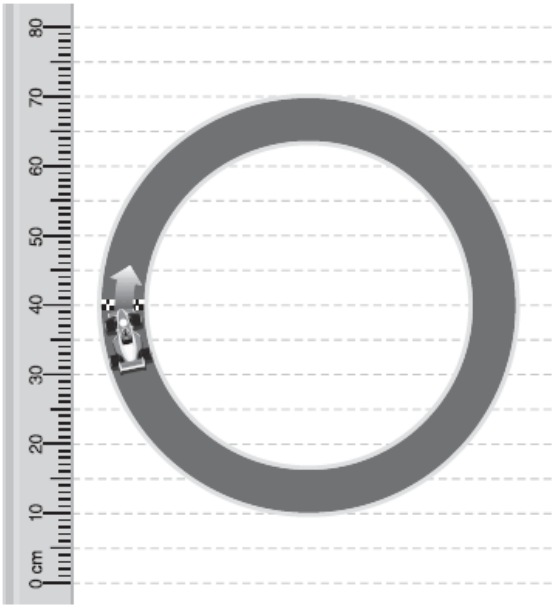
Math 30-2 Sinusoidal Functions Lesson 5 Practice Questions [Solutions at end]

Use the following information to answer the first question.

An electric toy car is travelling around a circular track at a constant speed. A ruler is positioned beside the track as shown in the graph below.



The position of the car, measured in cm with a ruler, can be modelled by the sinusoidal function, $p = 30\sin\left(\frac{\pi}{4}t\right) + 40$, where p is the position of the car in cm, and t is the time elapsed in seconds.

1. *The equation of the midline is y = _i_, and the time that it takes for the car to travel twice around the track is _ii_.*

The statement above is completed by the information in row

ROW	i	ii
A	40	16
B	40	8
C	30	16
D	30	8

Use the following information to answer the next question.

The sunrise time for a particular Western Canadian city can be modelled by the sinusoidal regression function

$$S = 1.45\sin(0.0172d + 1.51) + 6.6$$

where S is the sunrise time in hours after midnight, and d is the number of days since the beginning of the year (Jan. 1 = 1, Jan. 2 = 2, and so on)

2. According to the sinusoidal regression function, the sunrise time on June 16 (day 167), to the nearest hundredth, can be written in the form, K.MN, where K, M, and N are integers. The values of K, M, and N, respectively, are ___, ___, and ___.

Use the following information to answer the next question.

A Ferris wheel at a local carnival has a diameter of 46 feet and the minimum height above the ground reached by a seat is 5 feet.

When discussing the graph of a sinusoidal function that models the height of the seat above the ground during the ride, six students made the following statements.

Statement 1	The maximum value is 51.
Statement 2	The maximum value is 46.
Statement 3	The amplitude is 46.
Statement 4	The amplitude is 23.
Statement 5	The median is 23.
Statement 6	The median is 28.

3. The true statements are

A) 1, 4, 6 B) 1, 4, 5 C) 2, 3, 5 D) 2, 3, 6

Use the following information to answer the next question.

The table below shows the highest daily temperatures (in degrees Fahrenheit) averaged over the month for Austin Texas.

Month	^{0}F	**Month**	^{0}F
January	62	July	96
February	65	August	97
March	72	September	91
April	80	October	82
May	87	November	71
June	92	December	63

4. A) Determine the sinusoidal regression equation in the form $y = a \sin(bx + c) + d$. Round the values of *a*, *b*, *c*, and *d*, to three decimals.

   B) What percent of the year (to the nearest whole number) is the temperature in Austin Texas below 66^0 F?

Use the following information to answer the next question.

The height of a tide, in metres, in a particular harbour can be modelled by the sinusoidal function

$$h(t) = 5.5 \sin(0.51t + 1.57) + 8$$

where t represents the number of hours after midnight on a particular day.

5. To the nearest minute, the second earliest time after midnight on that day that the height of the tide is at its minimum depth is

   A) 6:16 B) 6:10 C) 18:29 D) 18:48

Use the following information to answer the next question.

The height above the ground of a rider on a Ferris wheel can be modelled by the sinusoidal function

$$h(t) = 7.25 \sin(1.42t - 1.57) + 10.75$$

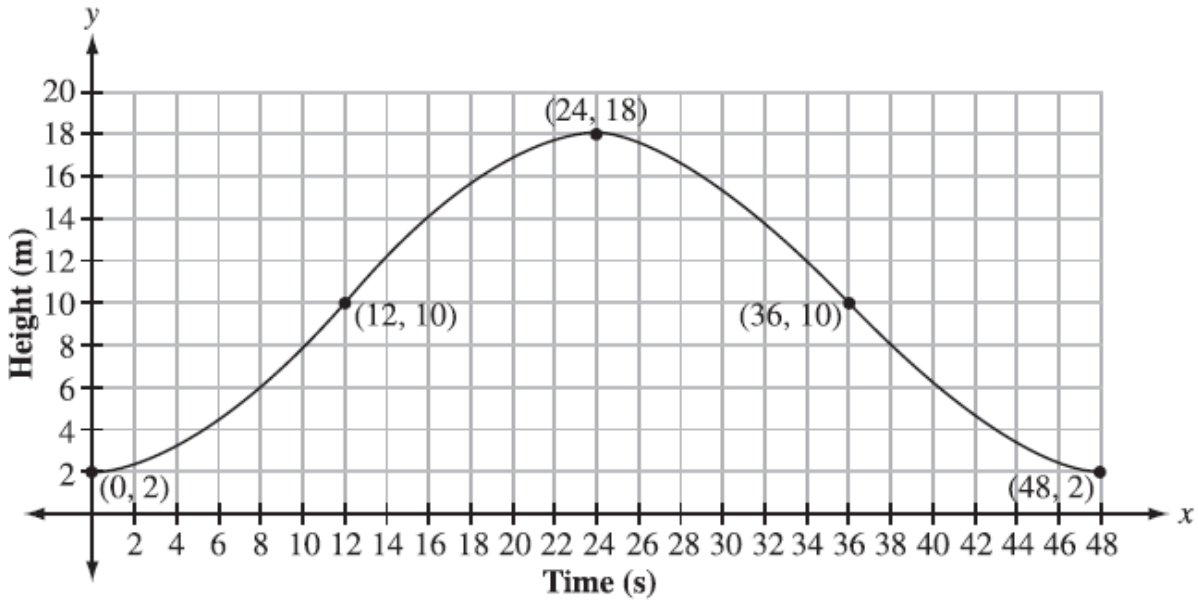
where h is the height of the rider above the ground, in metres, and t is the time in minutes, after the ride starts.

6. Based on the sinusoidal function, the maximum height of the rider above the ground is

A) 7.25 m B) 10.75 m C) 18 m D) 21 m

Use the following information to answer the next question.

Consider the graph below.



Possible Answers

Choice #1	a = 8	**Choice #4**	k = 48	**Choice #7**	d = 8
Choice #2	a = 10	**Choice #5**	k = 24	**Choice #8**	d = 10
Choice #3	a = 18	**Choice #6**	k = 2	**Choice #9**	d = 18

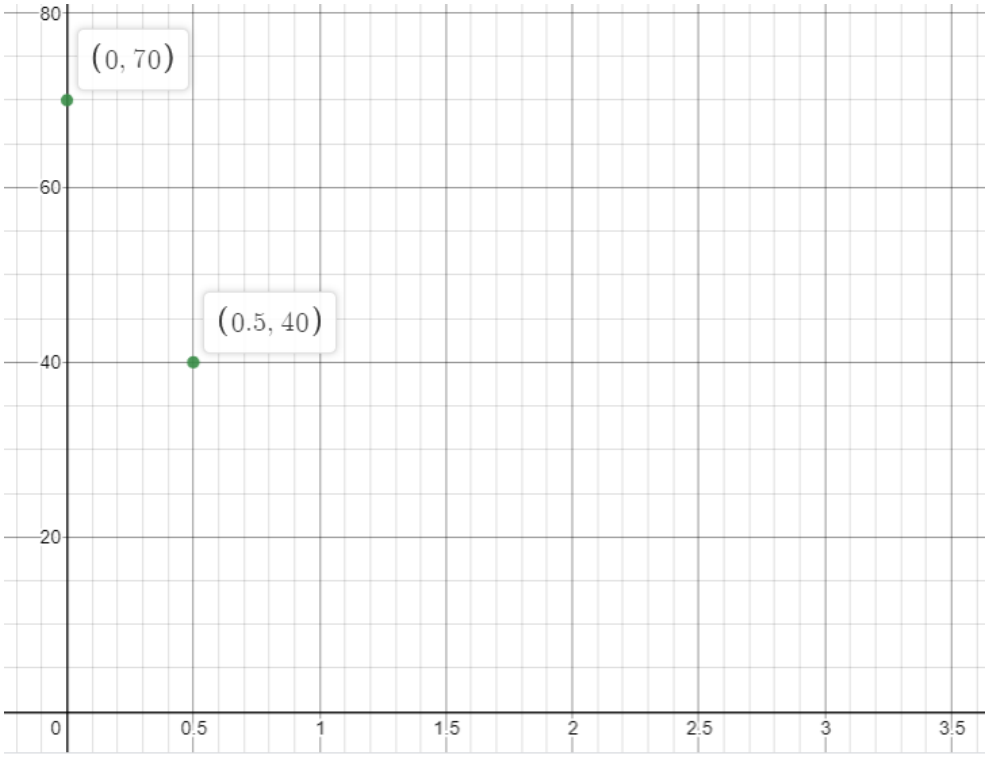
7. The graph can be written in the form, $y = a\sin\left(\frac{\pi}{k}x - 1.57\right) + d$

The value for *a* is choice number ____.
The value for *k* is choice number ____.
The value for *d* is choice number ____.

Use the following information to answer the next question.

Josh swings a pendulum back and forth. Its minimum height from the floor is 40 cm, and its maximum height is 70 cm. The pendulum falls from its highest point to the lowest point in 0.5 s.
Josh started by creating a table of value and he plotted the first 2 points.

Time	Height
0	70
0.5	40
1.0	
1.5	
2.0	
2.5	



8. a) Complete the table and plot the remaining points on the graph. Join the points.

   b) The sinusoidal function that models the data can be written in the form $y = a \cos(bx) + d$. Determine the values of a, b, and d.

   c) How high will the pendulum be after 7.25 s?

   d) What does the y-intercept represent?

Use the following information to answer the next question.

A Ferris wheel has a radius of 6.25 m and its centre is 7 m above the ground. A rider gets on a chair on the wheel at its lowest point and completes one full revolution in 2 min and 52 seconds (172 seconds).

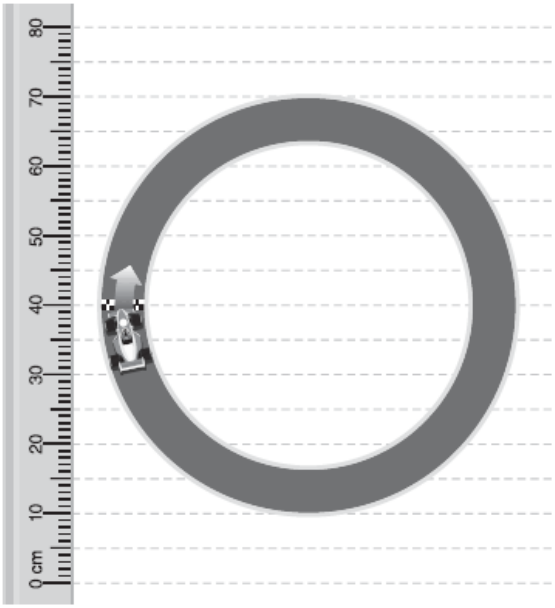
9. a) Determine a function of the form $y = a \bullet \sin(bx - 1.57) + d$.

   b) After one revolution, determine the length of time, to the nearest tenth, that the rider is at or above 12 m from the ground.

Math 30-2 Sinusoidal Functions Lesson 5 Practice QuestionsSolutions

Use the following information to answer the first question.

An electric toy car is travelling around a circular track at a constant speed. A ruler is positioned beside the track as shown in the graph below.



The position of the car, measured in cm with a ruler, can be modelled by the sinusoidal function, $p = 30\sin\left(\frac{\pi}{4}t\right) + 40$, where p is the position of the car in cm, and t is the time elapsed in seconds.

1. *The equation of the midline is y = _i_, and the time that it takes for the car to travel twice around the track is _ii_.*

The statement above is completed by the information in row

ROW	i	ii
A	40	16
B	40	8
C	30	16
D	30	8

Solution

The equation of the midline is y = 40.

To determine the period, or the time it takes for the car to go once around the track, use the formula, $period = \frac{2\pi}{b}$. The value of b in the equation is $\frac{\pi}{4}$.

$$period = \frac{2\pi}{\frac{\pi}{4}}$$

$$period = \left(\frac{2\pi}{1}\right)\left(\frac{4}{\pi}\right)$$

Period = 8 seconds

To go once around the track takes 8 seconds, and it takes 16 seconds to go around the track twice.

The correct answer is A.

Use the following information to answer the next question.

The sunrise time for a particular Western Canadian city can be modelled by the sinusoidal regression function

$$S = 1.45\sin(0.0172d + 1.51) + 6.6$$

where S is the sunrise time in hours after midnight, and d is the number of days since the beginning of the year (Jan. 1 = 1, Jan. 2 = 2, and so on)

2. According to the sinusoidal regression function, the sunrise time on June 16 (day 167), to the nearest hundredth, can be written in the form, K.MN, where K, M, and N are integers. The values of K, M, and N, respectively, are _5_, _2_, and _3_.

Solution

Substitute d = 167 in the equation.

$S = 1.45 \sin(0.0172(167) + 1.51) + 6.6$

$S = 1.45 \sin(4.3824) + 6.6$

$S = 1.45 (-0.946\ldots) + 6.6$

$S = -1.371\ldots + 6.6$

$S = 5.228\ldots$

$S = 5.23$

The values of K, M, and N, respectively, are _5_, _2_, and _3__.

Use the following information to answer the next question.

A Ferris wheel at a local carnival has a diameter of 46 feet and the minimum height above the ground reached by a seat is 5 feet.

When discussing the graph of a sinusoidal function that models the height of the seat above the ground during the ride, six students made the following statements.

Statement 1	The maximum value is 51.
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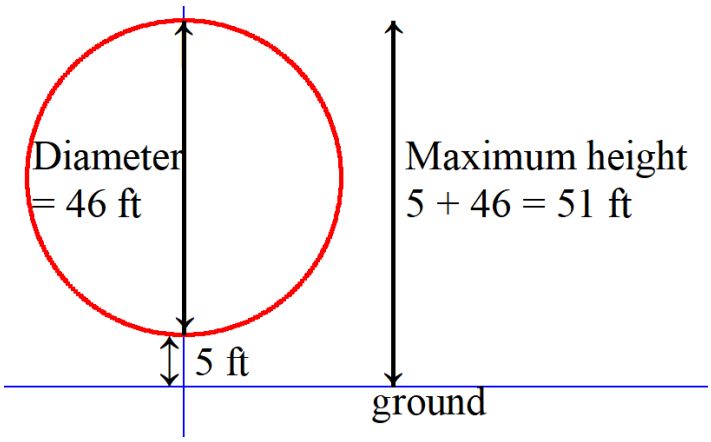
Statement 3	The amplitude is 46.
Statement 4	The amplitude is 23.
Statement 5	The median is 23.
Statement 6	The median is 28.

3. The true statements are

A) 1, 4, 6 B) 1, 4, 5 C) 2, 3, 5 D) 2, 3, 6

Solution

The maximum height is 51 ft.



The amplitude is:

$$\frac{\max value - \min value}{2}$$

$$\frac{51-5}{2}$$

= 23 ft

The median is:

$$\frac{\max value + \min value}{2}$$

$$\frac{51+5}{2}$$

= 28 ft

The correct answer is A.

Use the following information to answer the next question.

The table below shows the highest daily temperatures (in degrees Fahrenheit) averaged over the month for Austin Texas.

Month	^{0}F	**Month**	^{0}F
January	62	July	96
February	65	August	97
March	72	September	91
April	80	October	82
May	87	November	71
June	92	December	63

4. A) Determine the sinusoidal regression equation in the form $y = a \sin(bx + c) + d$. Round the values of a, b, c, and d, to three decimals.

Solution

Use the calculator by inputting the month number (1 – 12) in list one and the temperature in list 2.

Choose Stat, Calculate, Sin Regression.

Select L_1, L_2, Y_1 [Y_1 from pressing VARS, Y-VARS, enter]

By inserting Y_1, the equation will be pasted to the graphing y = menu.

$Y = 17.752 \sin (0.504x - 2.007) + 79.159$

B) What percent of the year (to the nearest whole number) is the temperature in Austin Texas below 66^0 F?

Solution

Graph $y_1 = 17.752 \sin(0.504x - 2.007) + 79.159$, and

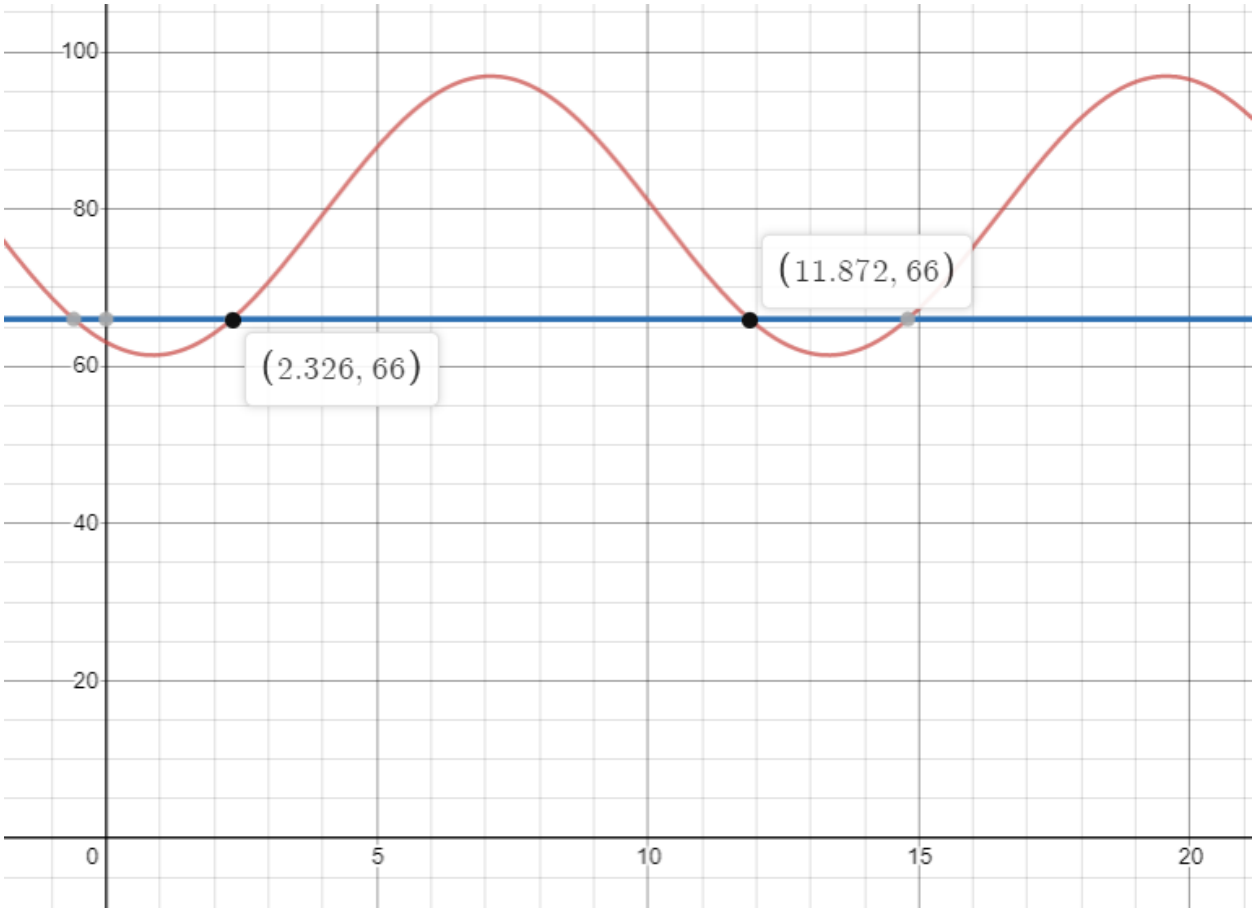
Graph $y_2 = 66$

The x-coordinates of the points of intersection are 2.326 and 11.872. These values represent months of the year. The difference between these values (9.546) represent the number of months above 66.

The percentage of time above 66 is $\frac{9.546}{12}$ or 79.55%.

The percentage of time below 66 is 100% - 79.55%, or 20.45%.

The percent of the year (to the nearest whole number) that the temperature in Austin Texas is below 66^0 F is 20%.



Use the following information to answer the next question.

The height of a tide, in metres, in a particular harbour can be modelled by the sinusoidal function

$$h(t) = 5.5 \sin(0.51t + 1.57) + 8$$

where t represents the number of hours after midnight on a particular day.

5. To the nearest minute, the second earliest time after midnight on that day that the height of the tide is at its minimum depth is

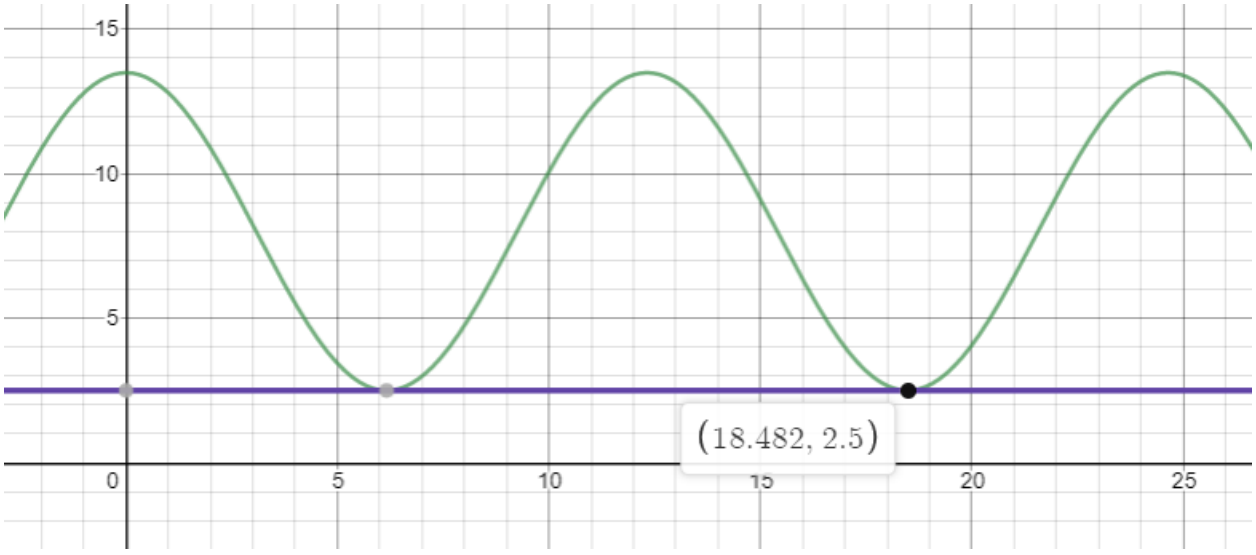
A) 6:16 B) 6:10 C) 18:29 D) 18:48

Solution

The minimum value is found by subtracting the amplitude (a = 5.5) from the d value (8).

The minimum value is 8 – 5.5, which is 2.5 m.

With the calculator, input $y_1 = 5.5 \sin(0.51t + 1.57) + 8$; and $y_2 = 2.5$



The x-coordinate of the second intersection point is 18.482.

Multiply (60) by (0.482) to find the number of minutes after 1800.

The number of minutes after 1800 is 28.92.

The correct answer is C.

Use the following information to answer the next question.

The height above the ground of a rider on a Ferris wheel can be modelled by the sinusoidal function

$$h(t) = 7.25 \sin(1.42t - 1.57) + 10.75$$

where h is the height of the rider above the ground, in metres, and t is the time in minutes, after the ride starts.

6. Based on the sinusoidal function, the maximum height of the rider above the ground is

A) 7.25 m B) 10.75 m C) 18 m D) 21 m

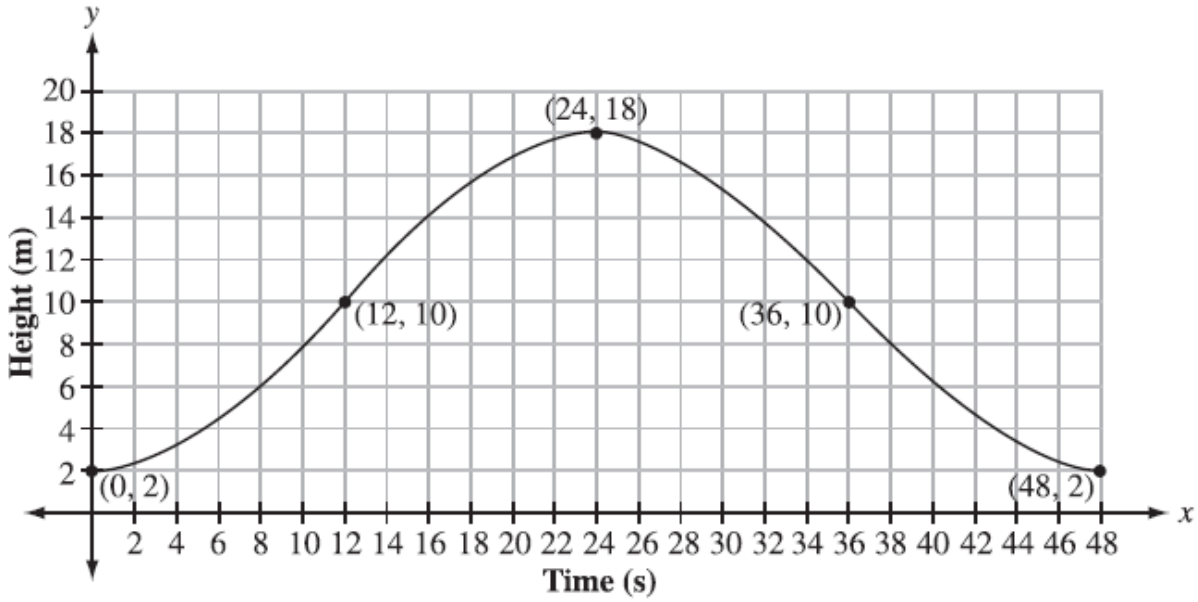
Solution

The equation of the midline (median) is y = 10.75. The amplitude is 7.25. The maximum value is determined by adding 7.25 to 10.75.

The maximum height of the rider above the ground is 18 m.

Use the following information to answer the next question.

Consider the graph below.



Possible Answers

Choice #1	a = 8	**Choice #4**	k = 48	**Choice #7**	d = 8
Choice #2	a = 10	**Choice #5**	k = 24	**Choice #8**	d = 10
Choice #3	a = 18	**Choice #6**	k = 2	**Choice #9**	d = 18

7. The graph can be written in the form, $y = a\sin\left(\frac{\pi}{k}x - 1.57\right) + d$

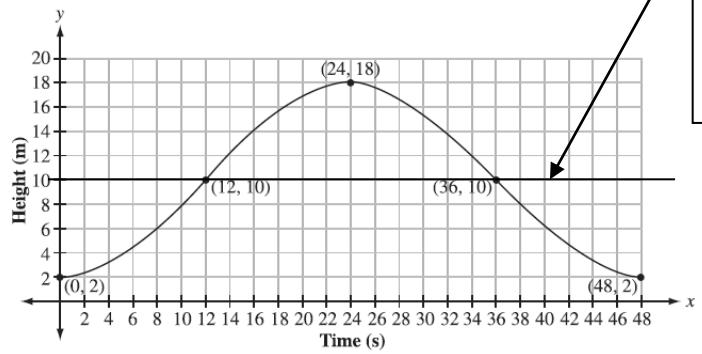
The value for *a* is choice number _1_.

The value for *k* is choice number _5_.

The value for *d* is choice number _8_.

The equation of the midline is y = 10. The value of d is 10.

Solution



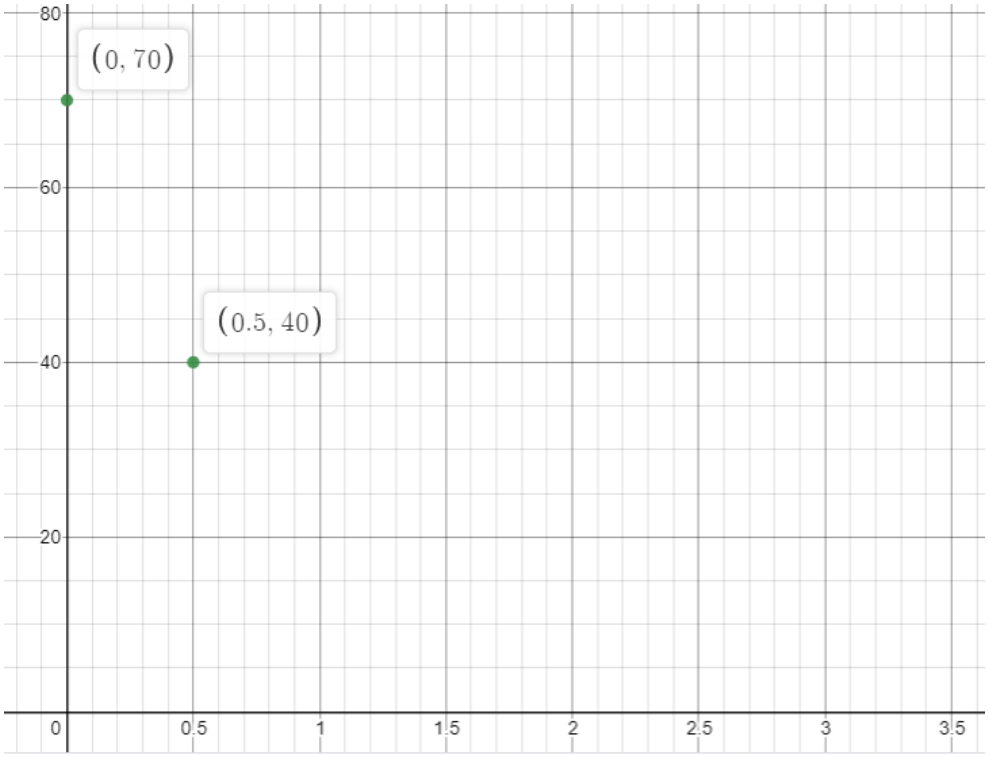
The distance from the midline to the maximum is 8 units. The value of a is 8.

The period is 48 sec. The value of b is $\frac{2\pi}{48}$ or $\frac{\pi}{24}$. The value of k is 24.

Use the following information to answer the next question.

Josh swings a pendulum back and forth. Its minimum height from the floor is 40 cm, and its maximum height is 70 cm. The pendulum falls from its highest point to the lowest point in 0.5 s.
Josh started by creating a table of value and he plotted the first 2 points.

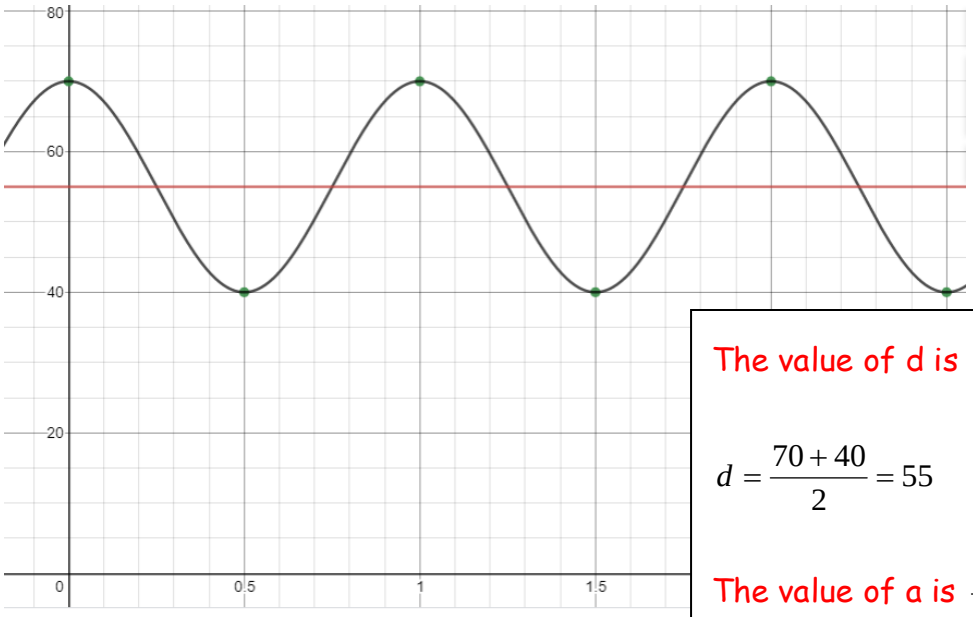
Time	Height
0	70
0.5	40
1.0	70
1.5	40
2.0	70
2.5	40



8. a) Complete the table and plot the remaining points on the graph. Join the points.

   b) The sinusoidal function that models the data can be written in the form $y = a \cos(bx) + d$. Determine the values of a, b, and d.

Solution



The value of d is $\frac{\max + \min}{2}$

$d = \frac{70 + 40}{2} = 55$

The value of a is $\frac{\max - \min}{2}$

The value of a is 15.

The period is 1 sec. The value of b is

$\frac{2\pi}{1} or 2\pi$

c) How high will the pendulum be after 7.25 s?

Solution

The equation of the sinusoidal function is y = 15 cos(2πx) + 55

Substitute x = 7.25.

y = 15 cos ((2π)(7.25) + 55

y = 15 (0) + 55

y = 55

After 7.25 seconds, the height of the pendulum will be 55 cm.

d) What does the y-intercept represent?

Solution

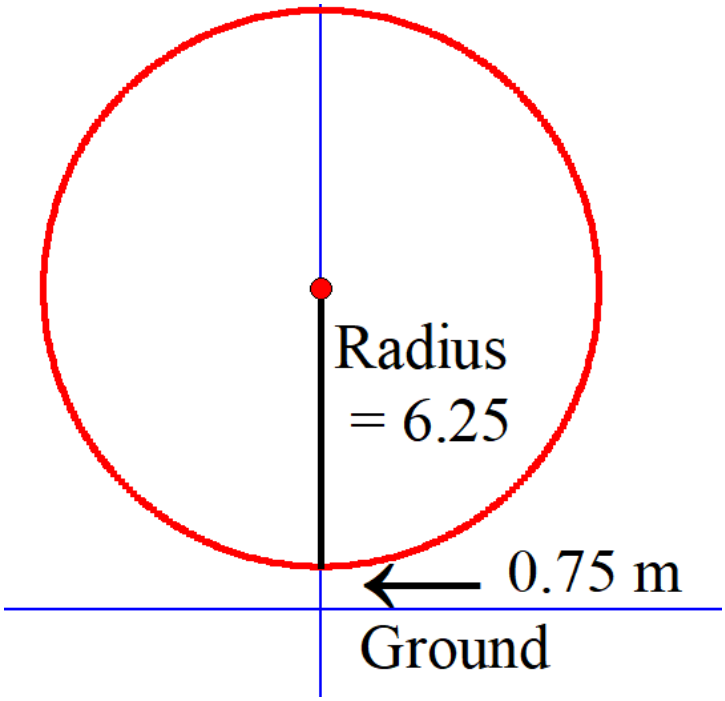
The y-intercept is (0, 70). This point means that at zero time (the moment the pendulum is released), the height of the pendulum is 70 cm.

Use the following information to answer the next question.

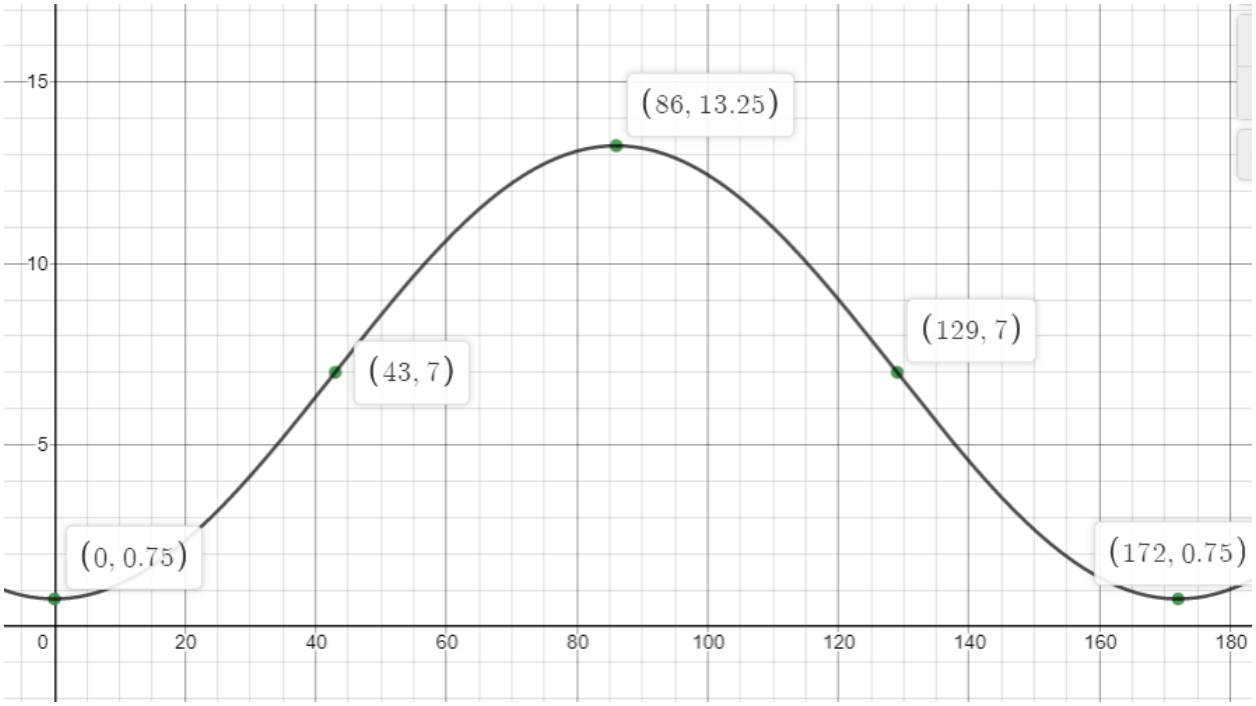
A Ferris wheel has a radius of 6.25 m and its centre is 7 m above the ground. A rider gets on a chair on the wheel at its lowest point and completes one full revolution in 2 min and 52 seconds (172 seconds).

9. a) Determine a function of the form $y = a \bullet \sin(bx - 1.57) + d$.

Solution



Time	Height
0	0.75
43	7
86	13.25
129	7
172	0.75



The equation of the midline is y = 7. The value of d is 7.

The amplitude is 6.25.

The period is 172. The value of b is $\frac{2\pi}{172} or \frac{\pi}{86}$.

The equation is y = 6.25 $\sin\left(\frac{\pi}{86}x - 1.57\right)$ + 7

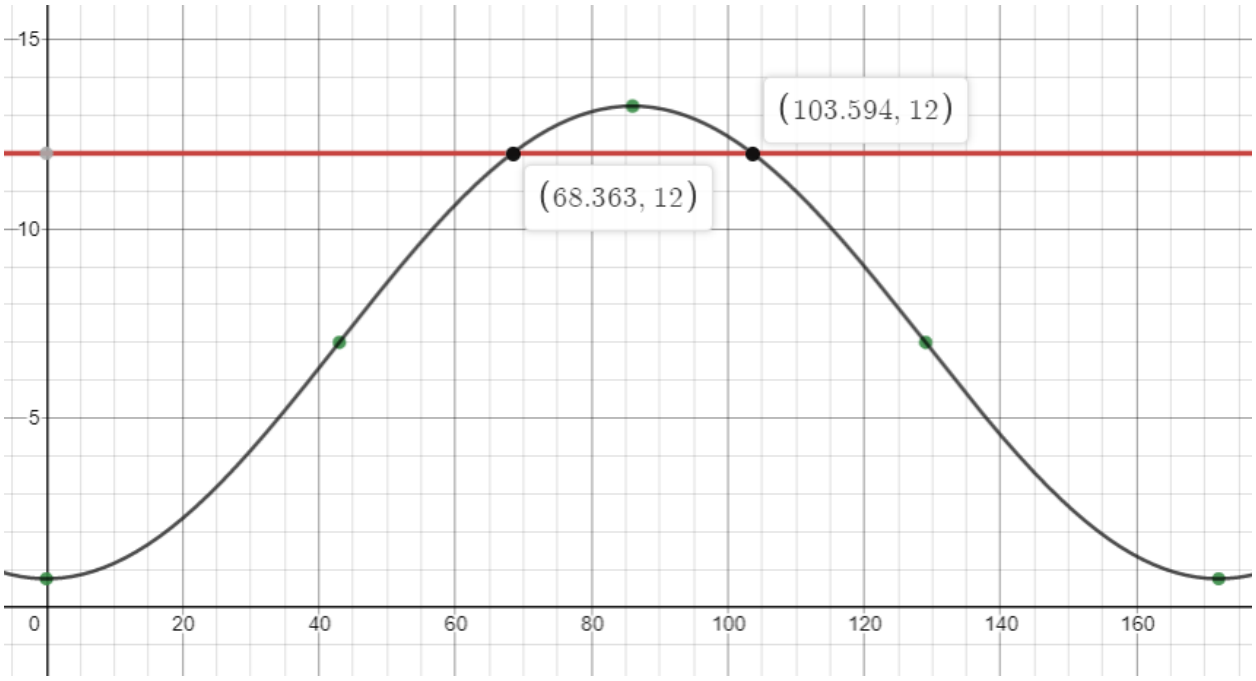
b) After one revolution, determine the length of time, to the nearest tenth, that the rider is at or above 12 m from the ground.

Solution

Graph $y_1 = 12$ and $y_2 = = 6.25 \sin\left(\frac{\pi}{86}x - 1.57\right) + 7$

Determine the intersection points.

The x-coordinates of the intersection points are 68.363 and 103.594. The difference between these values is 35.231.



After one revolution, the length of time, to the nearest tenth, that the rider is at or above 12 m from the ground is 35.2 seconds.